Algebra

Determinants

Definition

- The **determinant** is a scalar value that can be computed from a square matrix. It provides essential information such as:
- Whether a matrix is invertible
- The volume scaling factor of the linear transformation represented by the matrix
- For a **2×2 matrix**:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow \det(A) = ad - bc$$

- For a **3×3 matrix**:

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a(ei - fh) - b(di - fg) + c(dh - eg)$$

- This can be generalized for $n \times n$matrices using **Laplace expansion**.

Properties

- If $\det(A) \neq 0$, matrix is **invertible**
- If $\det(A) = 0$, matrix is **singular**
- Changing rows/columns affects the determinant:
  - Swapping rows changes sign
  - Adding a multiple of one row to another does not change determinant
  - Multiplying a row by scalar multiplies determinant by that scalar

Geometric Interpretation

- In 2D and 3D: The determinant gives the area or volume scaled by the transformation matrix.
- A determinant of zero means the transformation collapses the space (e.g., line or plane).

Properties of Determinants

Understanding the properties of determinants is crucial for simplifying computations and proving results in linear algebra.

Key Properties:

1. **Determinant of Identity Matrix**

$$\det(I_n) = 1$$

- For any identity matrix of order n, the determinant is 1.

2. Determinant of a Triangular Matrix

- For any upper or lower triangular matrix, or even a diagonal matrix, the determinant is the product of the diagonal elements.
- Example:

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 5 & 4 \\ 0 & 0 & 6 \end{bmatrix} \Rightarrow \det(A) = 3 \times 5 \times 6 = 90$$

3. Row or Column Swap

- If two rows (or columns) of a matrix are interchanged, the sign of the determinant changes:

$$\det(B) = -\det(A)$$

4. Identical Rows or Columns

- If two rows or two columns are identical, then:

$$\det(A) = 0$$

5. Row or Column of Zeros

- If any row or column consists entirely of zeros, then:

$$\det(A) = 0$$

6. Multiplying a Row or Column by a Scalar

- If a row (or column) is multiplied by a scalar k, the determinant is also multiplied by k.

$$\text{If one row: } \det(B) = k \cdot \det(A) \text{If two rows: } \det(B) = k^2 \cdot \det(A)$$

7. Adding a Multiple of One Row to Another

- This **does not** change the value of the determinant.
- This is useful in row operations.

8. Determinant of a Matrix and Its Transpose

$$\det(A^T) = \det(A)$$

Product of Determinants

- This property relates the determinant of a product of matrices to the determinants of the individual matrices.

Statement:

- If $A$and $B$are both square matrices of the same order, then:

$$\det(AB) = \det(A) \cdot \det(B)$$

Implications:

- It simplifies large matrix evaluations.
- Helps in proving results like:

$$\det(A^{-1}) = \frac{1}{\det(A)} \text{(when } A \text{ is invertible)}$$

Example:

- Let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$$

- Then:
  - $\det(A) = (1)(4) - (2)(3) = -2$
  - $\det(B) = (2)(2) - (0)(1) = 4$
  - $\det(AB) = \det(A) \cdot \det(B) = (-2)(4) = -8$
- You can verify that multiplying the matrices and computing the determinant gives the same result.

Introduction to Matrices

Definition

- A **matrix** is a rectangular array of numbers (or functions) arranged in rows and columns.
- It is often enclosed within square or round brackets.
- Matrices are used to represent systems of linear equations, transformations in space, and more.

Notation

- A matrix with m rows and n columns is called an **m × n** matrix, or a matrix of order m × n.
- **For example:**

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

  - is a **3 × 2** matrix.
- Each element in a matrix is denoted by a_{ij}, where:
  - i = row number
  - j = column number

Applications

- Matrices are foundational in:
  - Solving linear equations
  - Computer graphics
  - Quantum mechanics
  - Economics and statistics
  - Pharmacokinetics (e.g., compartment models)

Types of Matrices

Matrices come in various types, each with unique properties and applications:

A. Row Matrix

- Only one row.
- Example: $\begin{bmatrix} 2 & -3 & 5 \end{bmatrix}$

B. Column Matrix

- Only one column.

- Example: $\begin{bmatrix} 4 \\ -1 \\ 7 \end{bmatrix}$

C. Square Matrix

- Number of rows = number of columns.
- Example: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

D. Diagonal Matrix

- Square matrix with all non-diagonal elements zero.
- Example: $\begin{bmatrix} 5 & 0 \\ 0 & -3 \end{bmatrix}$

E. Scalar Matrix

- Diagonal matrix with all diagonal elements equal.
- Example: $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

F. Identity Matrix

- Square matrix with 1s on the diagonal and 0s elsewhere.
- Example: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

G. Zero/Null Matrix

- All elements are zero.
- Example: $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

H. Symmetric Matrix

- $A = A^T$, i.e., mirror image across the diagonal.
- Example: $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

I. Skew-Symmetric Matrix

- $A^T = -A$
- Diagonal elements are always zero.
- Example: $\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$

J. Upper Triangular Matrix

- All elements below the main diagonal are zero.
- Example: $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

First Hope

FREE PREVIEW

Click here to visit our website

PHARMACY PREMIUM NOTES

Complete subject and semesters are available on FirstHope app only.

Scan or click here

Click here to get the app >

Available on Google Play

JOIN THE FIRSTHOPE COMMUNITY

Join our Telegram channel

Click here to join Telegram >

Scan or click here

@FirstHopeNotes

Instagram

@firsthope.official

Click here to follow >

YouTube channel

FirstHope-Pharmacy / Live streams

Click here to watch >

WhatsApp channel

Follow FirstHope on WhatsApp

Click here to join >

FirstHope website

firsthope.co.in

Click here to visit >

B.Pharm / D.Pharm / M.Pharm / Pharm.D

Preview notes. Complete subjects and semesters in the FirstHope app.