Trigonometry

Sides and Angles of a Triangle

Basics of a Triangle

What is a Triangle?

- A **triangle** is a polygon with **three sides and three angles**. The sum of the internal angles of any triangle is always:

$$\angle A + \angle B + \angle C = 180^\circ$$

Each triangle has:

- **Three sides** (denoted as a, b, c)
- **Three angles** (denoted as A, B, C)
  - Angle $A$is opposite side a
  - Angle $B$is opposite side b
  - Angle $C$is opposite side c

Classification of Triangles

A. Based on Sides

- **Equilateral Triangle**: All three sides are equal and all angles are 60°.
- **Isosceles Triangle**: Two sides are equal; base angles are equal.
- **Scalene Triangle**: All three sides and all three angles are different.

B. Based on Angles

- **Acute Triangle**: All angles are less than 90°.
- **Right Triangle**: One angle is exactly 90°.
- **Obtuse Triangle**: One angle is greater than 90°.

- In trigonometry, we often focus especially on **right triangles**, but general triangles are also addressed through special rules and laws.

Elements of a Triangle

- Let's denote a triangle as $\triangle ABC$with corresponding sides:
  - Side a: opposite angle A
  - Side b: opposite angle B
  - Side c: opposite angle C
- Key elements:

- **Angle**: The amount of rotation between two intersecting sides.
- **Side**: The length of the segment connecting two vertices.
- **Height (Altitude)**: Perpendicular segment from a vertex to the opposite side.
- **Base**: Usually referred to one side (commonly side b) used as a reference.

Angle Notation and Measurement

- Angles can be measured in:
  - **Degrees (°)**: 360° in a full circle.
  - **Radians**: πradians = 180°.
- Conversion:

$$\text{Radians} = \frac{\pi}{180} \times \text{Degrees}$$

Basic Trigonometric Ratios (Right-Angled Triangle)

- For a right triangle with angle θ, and sides defined as:
  - **Hypotenuse**: Side opposite the right angle (longest side).
  - **Opposite**: Side opposite the angle θ.
  - **Adjacent**: Side next to angle θ(not hypotenuse).
- Then the **basic trigonometric ratios** are:

Ratio	Definition	Formula
Sine ($\sin\theta$)	Opposite to Hypotenuse	$\frac{\text{Opposite}}{\text{Hypotenuse}}$
Cosine ($\cos\theta$)	Adjacent to Hypotenuse	$\frac{\text{Adjacent}}{\text{Hypotenuse}}$
Tangent ($\tan\theta$)	Opposite to Adjacent	$\frac{\text{Opposite}}{\text{Adjacent}}$

- **Reciprocal Ratios:**

Ratio	Formula
Cosecant ($\csc\theta$)	$\frac{1}{\sin\theta} = \frac{\text{Hypotenuse}}{\text{Opposite}}$
Secant ($\sec\theta$)	$\frac{1}{\cos\theta} = \frac{\text{Hypotenuse}}{\text{Adjacent}}$
Cotangent ($\cot\theta$)	$\frac{1}{\tan\theta} = \frac{\text{Adjacent}}{\text{Opposite}}$

Pythagoras Theorem (Right Triangle Only)

$$\text{Hypotenuse}^2 = \text{Opposite}^2 + \text{Adjacent}^2$$
$$c^2 = a^2 + b^2$$

- This fundamental relationship is used to find unknown sides in right-angled triangles.

Trigonometric Identities

- These are universally true expressions involving trigonometric ratios:

1. **Pythagorean Identity:**

$$\sin^2\theta + \cos^2\theta = 1$$

2. **From this, we derive:**

$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

Angle Relationships and Quadrants

- **Complementary Angles:**

$$\sin(90^\circ - \theta) = \cos\theta, \cos(90^\circ - \theta) = \sin\theta$$

- **Supplementary Angles:**

$$\sin(180^\circ - \theta) = \sin\theta, \cos(180^\circ - \theta) = -\cos\theta$$

- **ASTC Rule (All Students Take Calculus):**
  - In the 1st Quadrant: All ratios are positive.
  - In the 2nd Quadrant: Only sine and cosecant are positive.
  - In the 3rd Quadrant: Only tangent and cotangent are positive.
  - In the 4th Quadrant: Only cosine and secant are positive.

Trigonometric Ratios of Standard Angles

Angle	sin	cos	tan
0°	0	1	0
30°	1/2	$\sqrt{3}/2$	$1/\sqrt{3}$
45°	$\sqrt{2}/2$	$\sqrt{2}/2$	1
60°	$\sqrt{3}/2$	1/2	$\sqrt{3}$
90°	1	0	Undefined

- These values are foundational for solving triangles efficiently.

Definition of a Triangle

- A triangle is a closed figure made up of three sides and three angles.
- The sum of the internal angles of any triangle is always 180 degrees.

Types of triangles (based on angles):

- **Acute Triangle**: All angles < 90°
- **Right Triangle**: One angle = 90°
- **Obtuse Triangle**: One angle > 90°

Types of triangles (based on sides):

- **Equilateral Triangle**: All sides equal, all angles = 60°
- **Isosceles Triangle**: Two sides equal
- **Scalene Triangle**: All sides different

Basic Trigonometric Ratios (Right-Angled Triangle)

- Let's consider a **right triangle** with:
  - Angle θ
  - Hypotenuse = longest side (opposite the right angle)
  - Opposite = side opposite to θ
  - Adjacent = side next to θ(not the hypotenuse)
- The **primary trigonometric ratios** are:

Ratio Name	Formula	Description
Sine ($\sin\theta$)	$\frac{\text{Opposite}}{\text{Hypotenuse}}$	Ratio of opposite side to hypotenuse
Cosine ($\cos\theta$)	$\frac{\text{Adjacent}}{\text{Hypotenuse}}$	Ratio of adjacent side to hypotenuse
Tangent ($\tan\theta$)	$\frac{\text{Opposite}}{\text{Adjacent}}$	Ratio of opposite side to adjacent

- There are also **reciprocal ratios**:

Ratio Name	Formula	Reciprocal Of
Cosecant ($\csc\theta$)	$\frac{1}{\sin\theta}$	Sine
Secant ($\sec\theta$)	$\frac{1}{\cos\theta}$	Cosine
Cotangent ($\cot\theta$)	$\frac{1}{\tan\theta}$	Tangent

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