Integral Calculus

Integration – Introduction, Definition, Standard Formulae

Introduction to Integration

- Integration is a fundamental concept in calculus.
- It is often described as the **reverse process of differentiation**, but more broadly, it refers to finding the **area under curves**, among many other applications.

There are two main types of integration:

1. **Indefinite integration**: Represents a family of functions (no bounds)
2. **Definite integration**: Represents a numerical value (area under a curve between two points)

Definition of Indefinite Integration

- If you are given a function $f(x)$, then its **indefinite integral** is written as:

$$\int f(x)\,dx$$

- This represents the **antiderivative** of $f(x)$, which is another function $F(x)$such that:

$$\frac{d}{dx}[F(x)] = f(x)$$

- We write:

$$\int f(x)\,dx = F(x) + C$$

- Where:
  - $F(x)$is any one antiderivative of $f(x)$
  - $C$is the **constant of integration** (since differentiating a constant gives 0)

Standard Integration Formulae

- Here are some commonly used standard formulae:

1. Power Rule:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C(n \neq -1)$$

2. Exponential Functions:

$$\int e^x\,dx = e^x + C$$

$$\int a^x\,dx = \frac{a^x}{\ln a} + C(a > 0, a \neq 1)$$

3. Logarithmic Function:

$$\int \frac{1}{x}\,dx = \ln\,|\,x\,|\,+C$$

4. Trigonometric Functions:

$$\int \sin\,x\,dx = -\cos\,x + C$$

$$\int \cos\,x\,dx = \sin\,x + C$$

$$\int \sec^2 x\,dx = \tan\,x + C$$

$$\int \csc^2 x\,dx = -\cot\,x + C$$

$$\int \sec\,x\tan\,x\,dx = \sec\,x + C$$

$$\int \csc\,x\cot\,x\,dx = -\csc\,x + C$$

Definite Integrals

Definition

- A **definite integral** is the integral of a function between two specified limits.
- It gives the **net area** under the curve of the function between the lower limit $a$and upper limit $b$on the x-axis.
- It is denoted as:

$$\int_a^b f(x)\,dx$$

- Where:
  - $f(x)$is the integrand.
  - $a$is the lower limit of integration.
  - $b$is the upper limit of integration.
  - dxindicates that the integration is with respect to variable x.

Fundamental Theorem of Calculus

- If $F(x)$is the **indefinite integral** of $f(x)$, i.e.,

$$F(x) = \int f(x)\,dx$$

- Then:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

- So, you:
  1. Find the **indefinite integral** $F(x)$
  2. Evaluate it at the **upper limit** b
  3. Subtract the value at the **lower limit** a

Example

$$\int_0^2 x^2\,dx$$

Step 1:

$$\int x^2 dx = \frac{x^3}{3}$$

Step 2:

- Evaluate from 0 to 2:

$$\frac{x^3}{3}\Big|_0^2 = \frac{2^3}{3} - \frac{0^3}{3} = \frac{8}{3}$$

Properties of Definite Integrals

1. **Reversal of limits:**

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

2. **Zero-width interval:**

$$\int_a^a f(x)dx = 0$$

3. **Additivity over intervals:**

$$\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$$

4. **Linearity:**

$$\int_a^b [af(x) + bg(x)]dx = a\int_a^b f(x)dx + b\int_a^b g(x)dx$$

- Definite integrals are especially important in **physics**, **engineering**, and **economics**, as they often represent total quantities over time, space, or other variables.

Method of Substitution

Purpose of Substitution

- The **substitution method** (also called **u-substitution**) is a powerful technique used to simplify complicated integrals by changing variables. It works similarly to the **chain rule** in differentiation, but in reverse.

Basic Idea

- Suppose you have an integral of the form:

$$\int f(g(x)) \cdot g'(x)\, dx$$

- Then, using substitution:
  - Let $u = g(x)$
  - Then $\frac{du}{dx} = g'(x) \Rightarrow du = g'(x)\, dx$
- This transforms the integral into:

$$\int f(u)\, du$$

- Which is typically easier to integrate.

Step-by-Step Process

1. **Choose a substitution**: Identify a part of the integrand that, when substituted, simplifies the expression.
2. **Compute du**: Differentiate your substitution to find du.
3. **Change all variables**: Rewrite the integral in terms of u and du.
4. **Integrate in u**
5. **Substitute back**: Replace u with the original expression.

Example

$$\int 2x \cdot \cos(x^2)\, dx$$

Step 1:

- Let $u = x^2 \Rightarrow du = 2x\, dx$
- Now the integral becomes:

$$\int \cos(u)\, du = \sin(u) + C$$

Step 2:

- Substitute back:

$$\sin(x^2) + C$$

Example

$$\int \frac{x}{x^2 + 1}\, dx$$

First Hope

FREE PREVIEW

Click here to visit our website

PHARMACY PREMIUM NOTES

Complete subject and semesters are available on FirstHope app only.

Scan or click here

Click here to get the app >

Available on Google Play

JOIN THE FIRSTHOPE COMMUNITY

Join our Telegram channel

Click here to join Telegram >

Scan or click here

@FirstHopeNotes

Instagram

@firsthope.official

Click here to follow >

YouTube channel

FirstHope-Pharmacy / Live streams

Click here to watch >

WhatsApp channel

Follow FirstHope on WhatsApp

Click here to join >

FirstHope website

firsthope.co.in

Click here to visit >

B.Pharm / D.Pharm / M.Pharm / Pharm.D

Preview notes. Complete subjects and semesters in the FirstHope app.