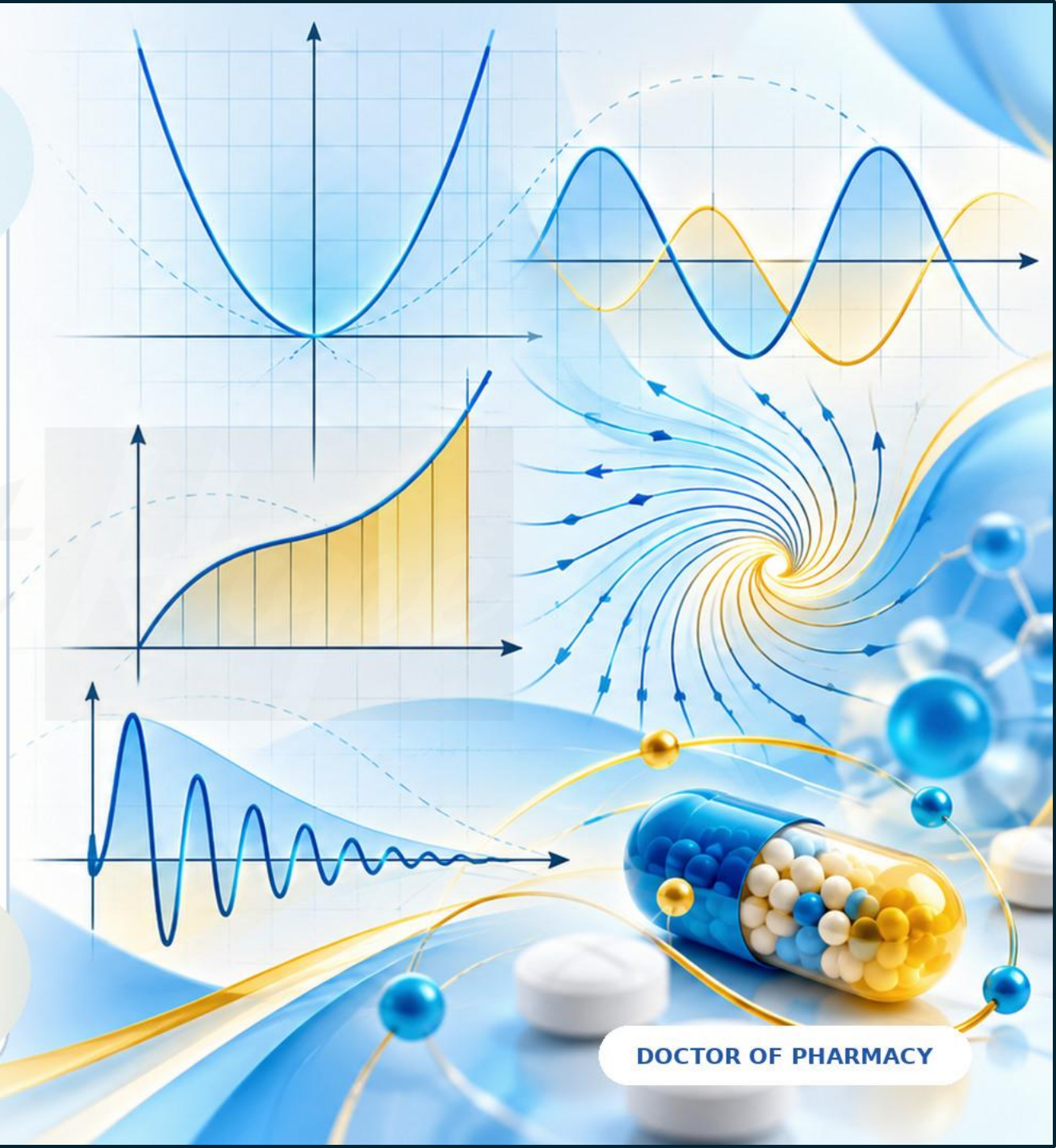
PHARM.D • YEAR 1

Remedial Mathematics

CHAPTER 7

Laplace Transform

PPT CHAPTER COVER



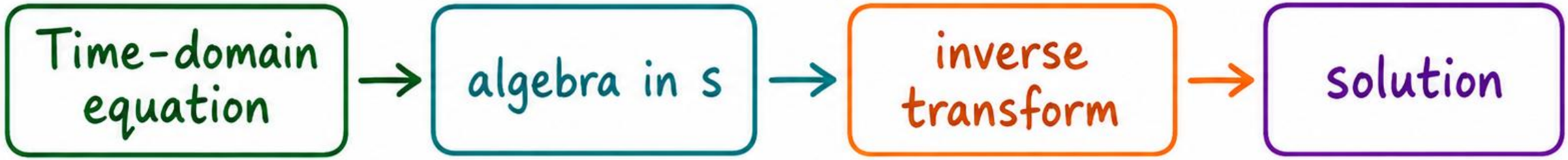
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Laplace transform: time to s-domain

The one-sided Laplace transform is:

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt.$$

- It uses the function for $t \geq 0$.
- $s = \sigma + i\omega$ is generally complex.
- The exponential kernel weights the function before integration.
- This is a weighted integral, not a normalised average.
- For suitable linear differential equations:



- Initial conditions can be included directly.

When does a Laplace transform exist?

1. **Regularity condition:**
   f is piecewise continuous on every finite interval in $[0, \infty)$.

2. **Exponential order condition:**
   For sufficiently large t, $|f(t)| \leq Me^{at}$ for some $M > 0$ and real a.

This is called exponential order.

Then the transform converges when

$$\mathrm{Re}(s) > a.$$

These conditions are sufficient, not necessary in every case.

Always state a region of convergence.

Region of convergence: $\mathrm{Re}(s) > a$.

For real s,
$\mathrm{Re}(s) > a$ simply becomes $s > a$.



Interval: (a, ∞)

Standard transforms: powers and exponentials

For real s in the stated range:

$$\mathcal{L}\{1\} = \frac{1}{s}, \quad s > 0.$$

$$\mathcal{L}\{t\} = \frac{1}{s^2}, \quad s > 0.$$

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}, \quad s > 0, \quad n = 0, 1, 2, \ldots$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}, \quad s > a.$$

EXAMPLE $\mathcal{L}\{t^2\} = \frac{2}{s^3}$.

The factorial belongs in the numerator.

Multiplication by an exponential shifts s in the transform.

Trigonometric and hyperbolic transforms

Trigonometric

$$\mathcal{L}\{\sin(at)\} = \frac{a}{s^2 + a^2}, \quad s > 0.$$

$$\mathcal{L}\{\cos(at)\} = \frac{s}{s^2 + a^2}, \quad s > 0.$$

Trigonometric denominators use $+a^2$.

Hyperbolic denominators use $-a^2$.

Hyperbolic

$$\mathcal{L}\{\sinh(at)\} = \frac{a}{s^2 - a^2}, \quad s > |a|.$$

$$\mathcal{L}\{\cosh(at)\} = \frac{s}{s^2 - a^2}, \quad s > |a|.$$

Sine and sinh have numerator a;

cosine and cosh have numerator s.

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