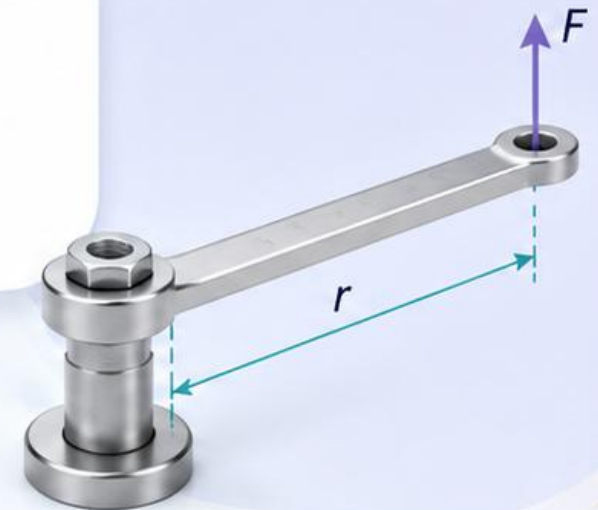
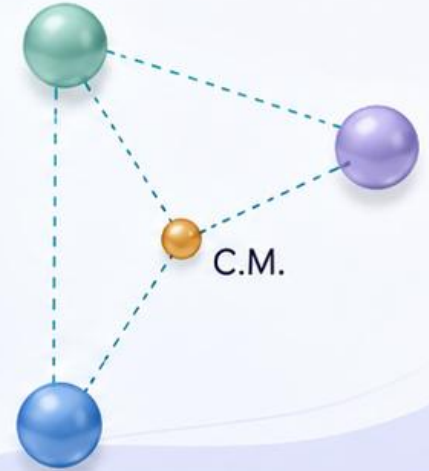
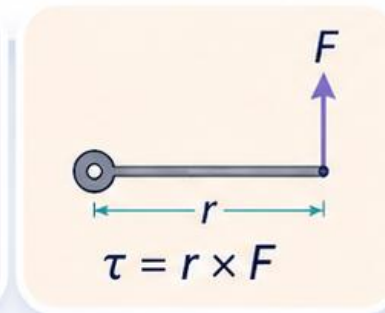
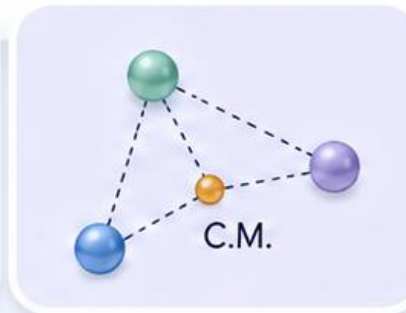


Chapter 6

System of Particles and Rotational Motion

Torque • Angular Motion • Centre of Mass

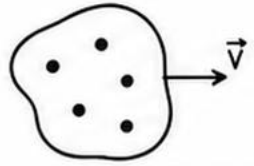


SYSTEMS OF PARTICLES & ROTATIONAL MOTION

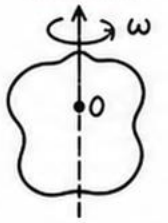
Extended bodies need two linked descriptions: motion of the centre of mass and motion about it.

① RIGID-BODY MOTION

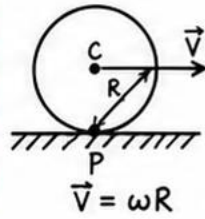
Pure translation



Fixed-axis rotation



Rolling

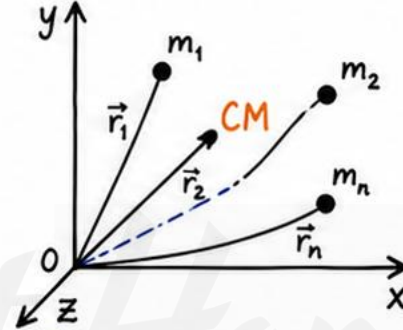


- shape and inter-particle distances remain fixed (ideal model).

② CENTRE OF MASS

$$\vec{R} = \frac{\sum_i m_i \vec{r}_i}{M}$$

$$M = \sum_i m_i$$

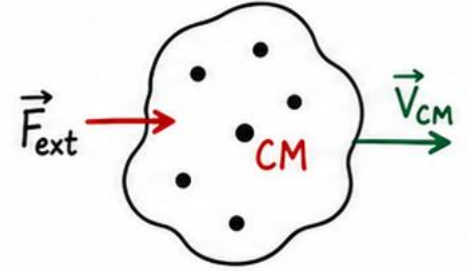


- mass-weighted position.

③ SYSTEM MOMENTUM

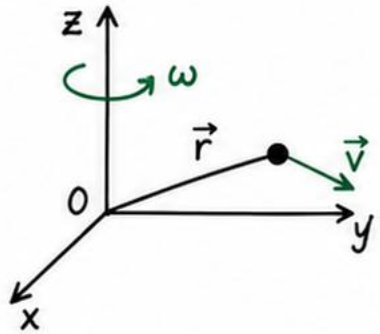
$$\vec{P} = M \vec{V}_{CM}$$

$$\vec{F}_{ext} = \frac{d\vec{P}}{dt}$$



- external force changes total momentum.

④ ANGULAR KINEMATICS



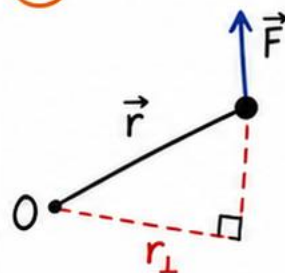
$$\theta = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt}$$

$$\vec{v} = \omega \times \vec{r}$$

- describe how orientation and angular speed change.

⑤ TORQUE & ANGULAR MOMENTUM



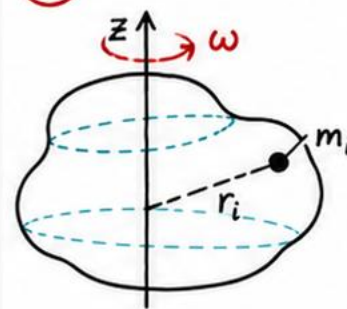
$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{l} = \vec{r} \times \vec{p}$$

$$(\vec{p} = m \vec{v})$$

- torque changes angular momentum.

⑥ ROTATIONAL DYNAMICS



$$\vec{I} = \sum_i m_i r_i^2$$

$$K_{rot} = \frac{1}{2} I \omega^2$$

$$\tau_z = I \alpha$$

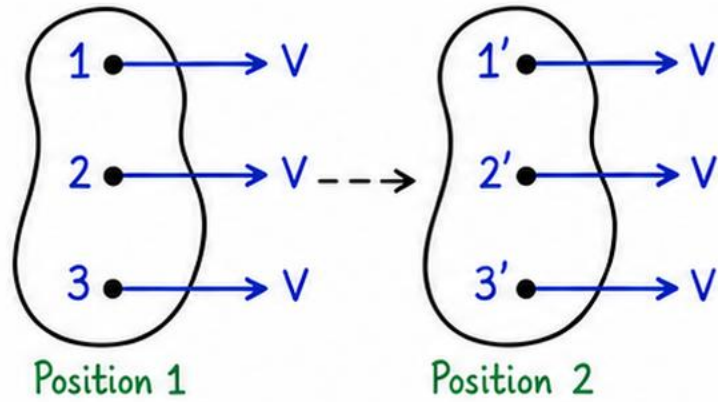
- relates torque, inertia and angular acceleration.

EXTERNAL FORCE controls COM motion • EXTERNAL TORQUE controls rotational motion

RIGID-BODY MOTION

An ideal rigid body keeps every distance between its particles unchanged; real bodies may be treated as rigid when deformation is negligible.

PURE TRANSLATION

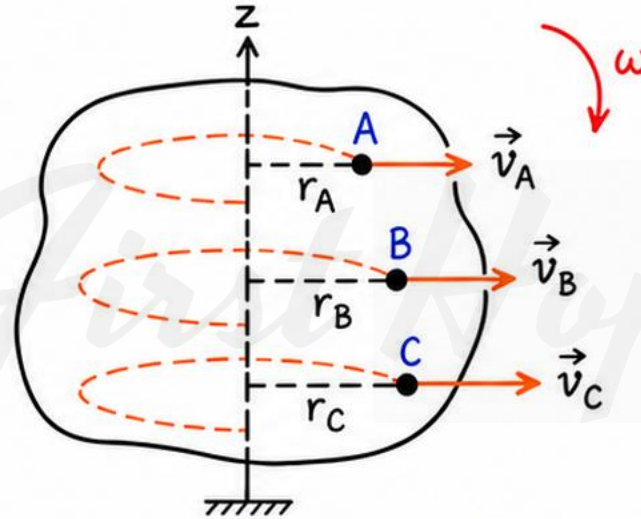


- All particles have the same velocity:

$$v_1 = v_2 = \dots = V_{CM}$$

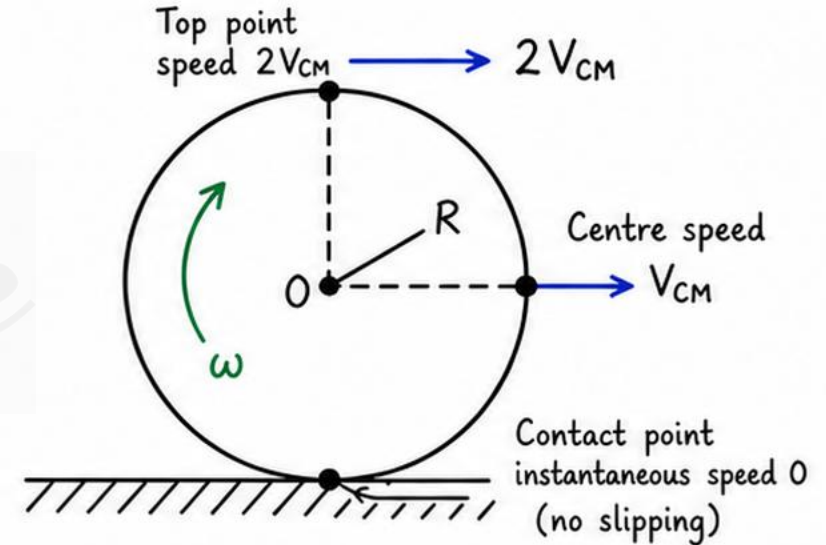
- All velocity vectors are equal and parallel (same direction and sense).
- Orientation does not change.

FIXED-AXIS ROTATION



- Points at different radii follow circles in planes perpendicular to the axis.
- All particles share the same angular velocity ω , but $v = \omega r_{\perp}$ depends on distance from the axis.
- Points on the axis have $r_{\perp} = 0 \Rightarrow v = 0$.

ROLLING = TRANSLATION + ROTATION

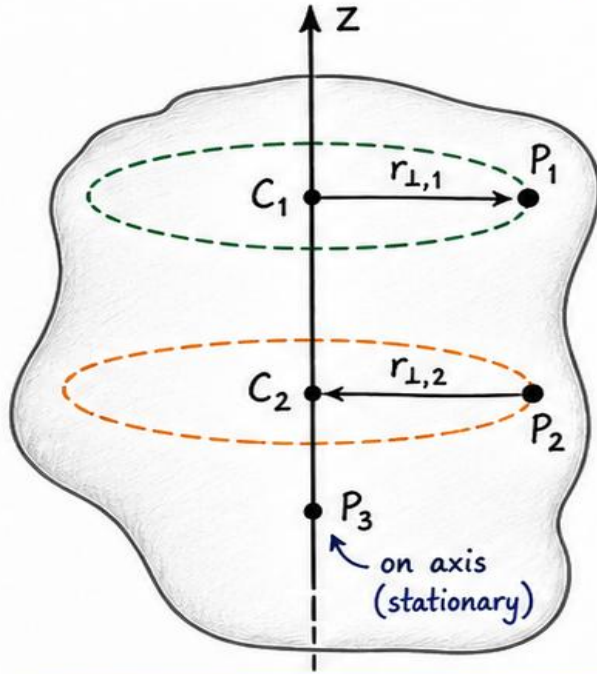


- The wheel rolls to the right without slipping.
- Top point speed = $2V_{CM}$ to the right.
- Contact point instantaneous speed = 0.
- No-slip condition: $V_{CM} = \omega R$.

Unfixed rigid body: translation or translation + rotation. Pivoted/fixed body: rotation about an axis.

ROTATION ABOUT AN AXIS

FIXED AXIS — GEOMETRY

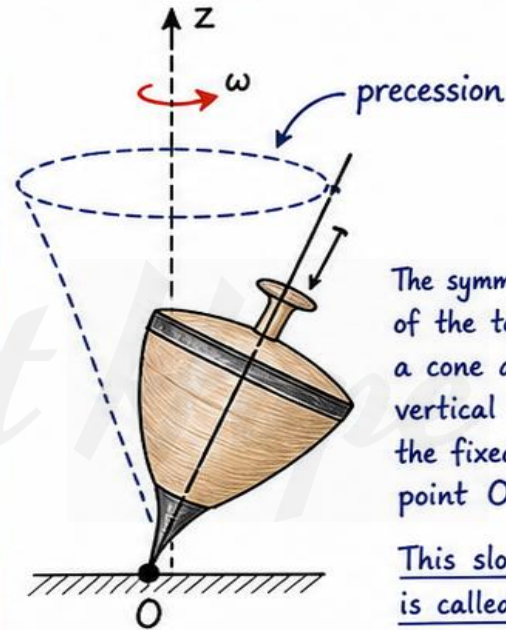


- The planes of the circles are perpendicular to the z -axis.
- C_1 and C_2 are the centres of the circles and lie on the axis.
- $r_{\perp,1}$ and $r_{\perp,2}$ are the perpendicular distances (radial distances) of P_1 and P_2 from the axis.

Every point follows a circle centred on the axis; one angular velocity ω describes the whole rigid body.

WHEN THE AXIS MOVES

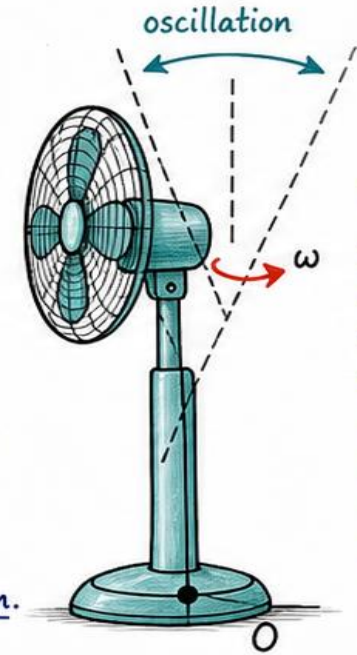
(a) Spinning top



The symmetry axis of the top sweeps a cone about the vertical through the fixed contact point O .

This slow motion is called precession.

(b) Pedestal fan



The axis is pivoted at O and oscillates to and fro. Throughout, the blades rotate about the axis with angular velocity ω .

With one line fixed, the axis is fixed. With only one point fixed, the instantaneous axis may change direction but passes through that point.

TAKEAWAY → Unless stated otherwise: rotation about a fixed axis.

CENTRE OF MASS – DISCRETE SYSTEMS

① Two particles on the x-axis

Consider two particles of masses m_1 and m_2 located on the x-axis at positions x_1 and x_2 respectively. Let C be the centre of mass at position X on the x-axis.

The position of C is given by
$$X = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

This is a mass-weighted mean. For positive masses, C lies between them and nearer to the heavier mass.

Equal masses: If $m_1 = m_2 = m$, then

$$X = \frac{mx_1 + mx_2}{m + m} = \frac{x_1 + x_2}{2} \quad (\text{midpoint})$$

② General n-particle system in three dimensions

Consider n particles of masses m_i at position vectors $\vec{r}_i = (x_i \hat{i} + y_i \hat{j} + z_i \hat{k})$, $i = 1, 2, \dots, n$ with respect to a fixed origin O.

Total mass:

$$M = \sum_{i=1}^n m_i$$

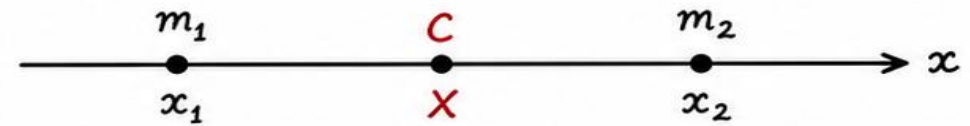
$$\begin{cases} X = \frac{\sum_{i=1}^n m_i x_i}{M}, & Y = \frac{\sum_{i=1}^n m_i y_i}{M}, \\ Z = \frac{\sum_{i=1}^n m_i z_i}{M} \end{cases}$$

In vector form,
$$\vec{R} = \frac{1}{M} \sum_{i=1}^n m_i \vec{r}_i \quad (\text{position vector of COM from O})$$

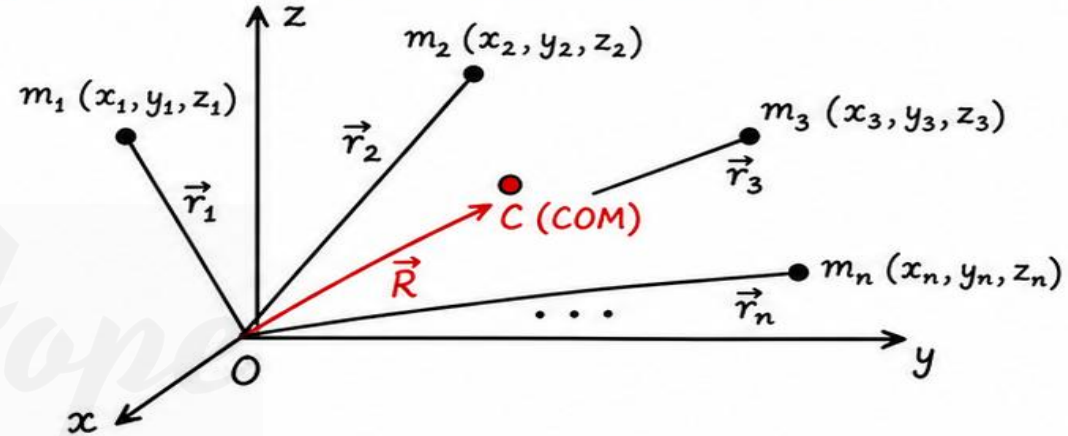
Quick check (dimensional and consistency):

Each coefficient $\frac{m_i}{M}$ is dimensionless and

$\sum_{i=1}^n \frac{m_i}{M} = \frac{1}{M} \sum_{i=1}^n m_i = \frac{M}{M} = 1$. Hence $\vec{R}$ is a weighted mean of position vectors and has the same dimensions as $\vec{r}_i$, i.e., length.



③ Vector diagram for a system of n particles



④ Origin at the centre of mass

Let $\vec{r}_i'$ be the position vector of mass m_i measured from the centre of mass C. Then

$$\sum_{i=1}^n m_i \vec{r}_i' = \vec{0} \quad (\text{Vector sum of position vectors about the COM is zero.})$$

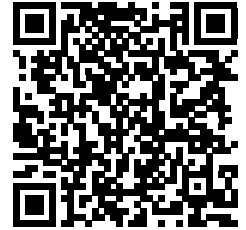


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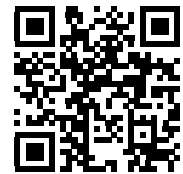


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