

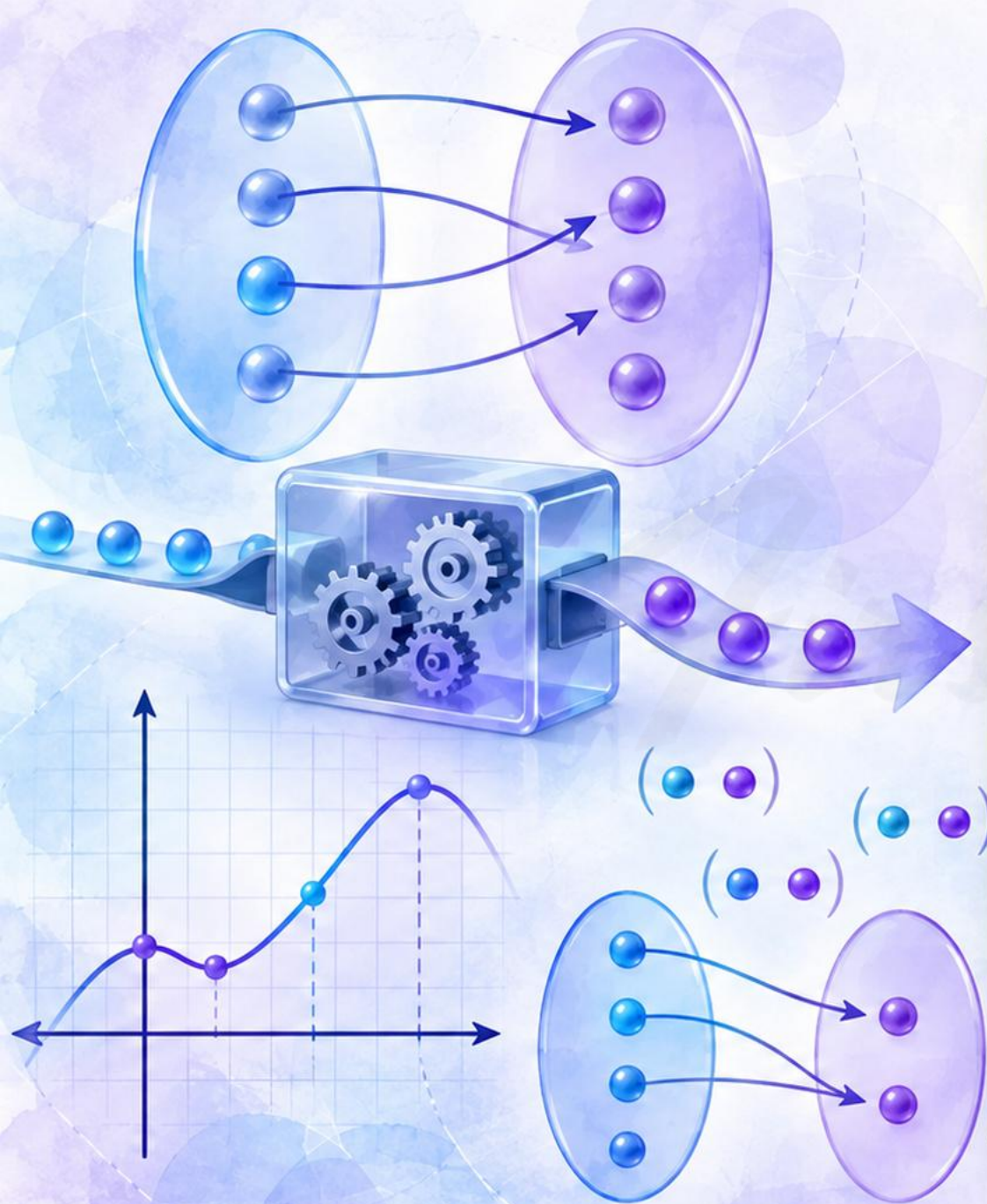
MATHEMATICS • CLASS 12

1

CHAPTER

Relations and Functions

CHAPTER NOTES



RELATIONS: BASIC IDEAS

What is a relation?

- A relation is any subset of the Cartesian product.
 $A \times B = \{(a, b) : a \in A, b \in B\}; R \subseteq A \times B.$
- A relation on A is a subset of $A \times A$.
 $(a, b) \in R \iff a R b.$
- This means a is related to b .
- A function is a special kind of relation.

EMPTY RELATION

- No element is related to any element.
 $R = \emptyset.$
- On $A = \{1, 2, 3, 4\}$, $a - b = 10$ gives no pairs.

Two ways to write it

$$A = \{1, 2, 3, 4\}; R = \{(a, b) \in A \times A : b = a + 1\}.$$

- 'Roster form':
 $R = \{(1, 2), (2, 3), (3, 4)\}.$
- 'Set-builder form' names the rule;
'roster form' lists the pairs.

UNIVERSAL RELATION

- Every element is related to every element.
 $R = A \times A.$
- On the same A , $|a - b| \geq 0$ is true for every pair.

Empty and universal relations are called trivial relations.

REFLEXIVE, SYMMETRIC & TRANSITIVE

These properties are checked for a relation R on a set A .

REFLEXIVE

Every element is related to itself.

$(a, a) \in R$ for every $a \in A$.



Check:

Look for every diagonal pair.
One missing (a, a) means not reflexive.

SYMMETRIC

If a is related to b , then b must be related to a .

$(a, b) \in R \Rightarrow (b, a) \in R$.



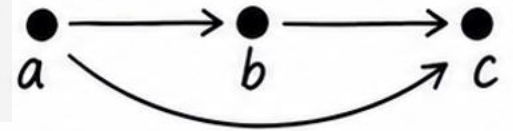
Check:

Every pair must have its reverse.

TRANSITIVE

If a is related to b and b to c , then a must be related to c .

$(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$.



Check:

Every two-step connection must also have a direct connection.

To disprove a property, one clear counterexample is enough.

CHECKING RELATION PROPERTIES

Perpendicular lines

Let L be all lines in a plane.

$$L_1 R L_2 \iff L_1 \perp L_2.$$

Not reflexive:

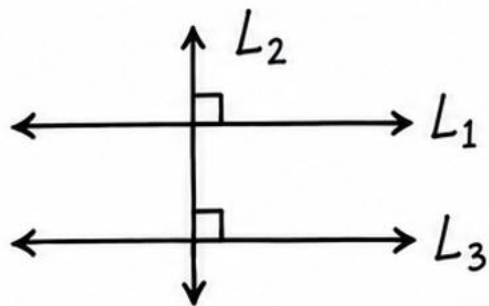
A line is not perpendicular to itself.

Symmetric:

$$L_1 \perp L_2 \Rightarrow L_2 \perp L_1.$$

Not transitive:

$L_1 \perp L_2$ and $L_2 \perp L_3$, but L_1 is not perpendicular to L_3 .



The horizontal lines are parallel, so the required direct relation fails.

A relation on three elements

$$A = \{1, 2, 3\}$$

$$R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}.$$

Reflexive:

All three diagonal pairs are present.

Not symmetric:

$$(1, 2) \in R, \text{ but } (2, 1) \notin R.$$

Not transitive:

$$(1, 2), (2, 3) \in R, \text{ but } (1, 3) \notin R.$$

Check the properties separately; one property does not guarantee the others.

EQUIVALENCE RELATIONS

An equivalence relation is reflexive, symmetric and transitive — ALL THREE are required.

Congruent triangles

$T_1 R T_2 \Leftrightarrow T_1 \cong T_2$, on the set of all triangles in a plane.

Reflexive: Every triangle is congruent to itself.

Symmetric: If $T_1 \cong T_2$, then $T_2 \cong T_1$.

Transitive: If $T_1 \cong T_2$ and $T_2 \cong T_3$, then $T_1 \cong T_3$.

Conclusion: Congruence is an equivalence relation.

Even difference on integers

$R = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} : 2 \text{ divides } a - b\}$.

Related integers have the same parity (both even or both odd).

1 Reflexive: $a - a = 0 = 2 \times 0$.

2 Symmetric: $a - b = 2k \Rightarrow b - a = -2k$.

3 Transitive: $a - b = 2k$, $b - c = 2m$;
 $a - c = (a - b) + (b - c) = 2(k + m)$.
 $k, m \in \mathbb{Z}$.

Each difference is divisible by 2, so R is an equivalence relation.

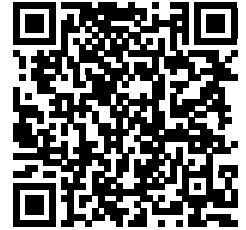


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