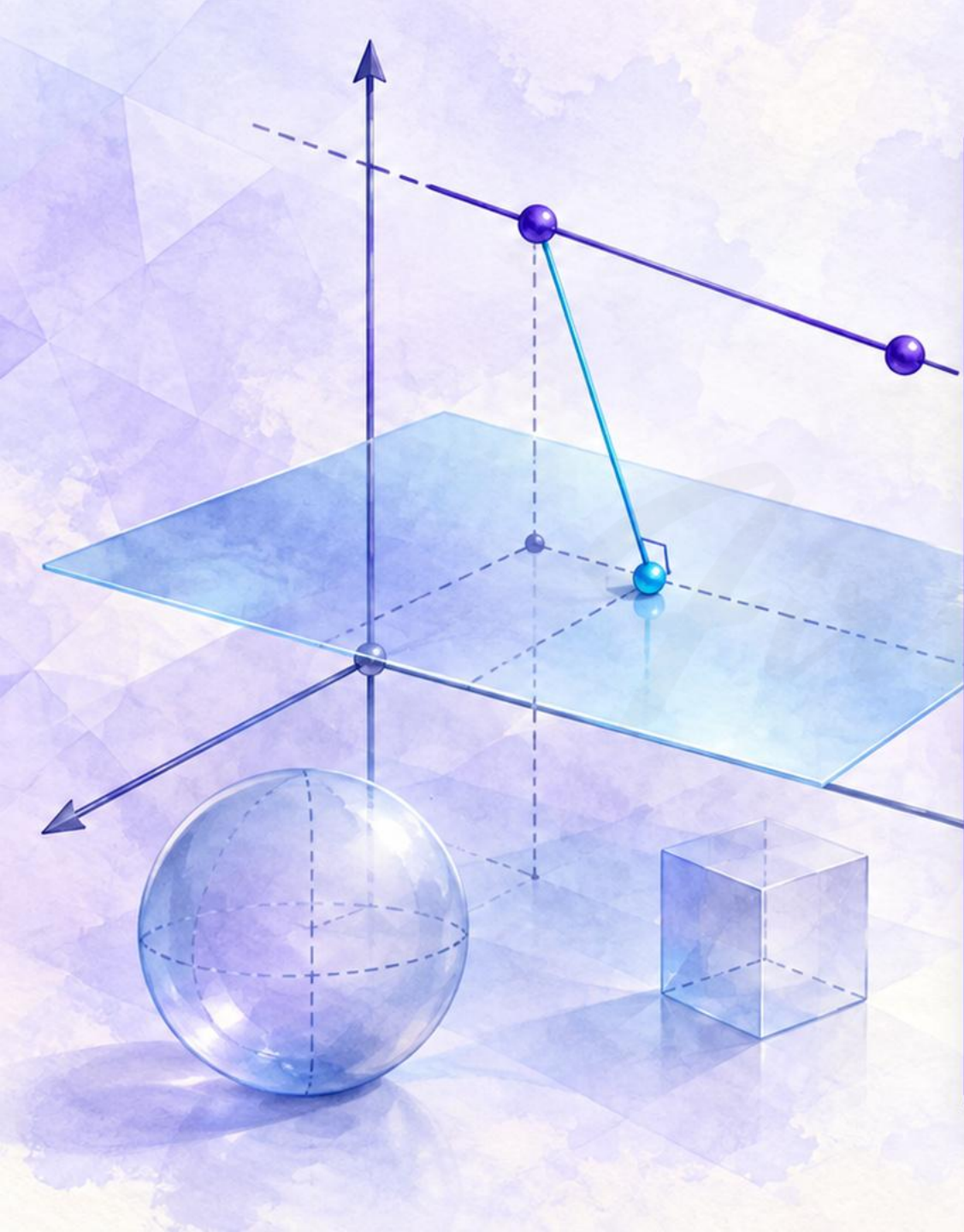


A 3D geometric diagram on the left side of the page. It features a vertical axis with an arrow at the top. A horizontal plane is shown, with a point on it connected by a dashed vertical line to a point on the axis. A line with two points passes through the plane. A sphere is at the bottom left, and a cube is at the bottom right. A line segment connects a point on the axis to a point on the plane, with a right-angle symbol at the intersection.

MATHEMATICS • CLASS 12

11

CHAPTER

Three Dimensional Geometry

CHAPTER NOTES

DIRECTION COSINES OF A LINE

Direction cosines are the cosines of the angles a directed line makes with the positive coordinate axes.

$$l = \cos \alpha, \quad m = \cos \beta, \quad n = \cos \gamma$$

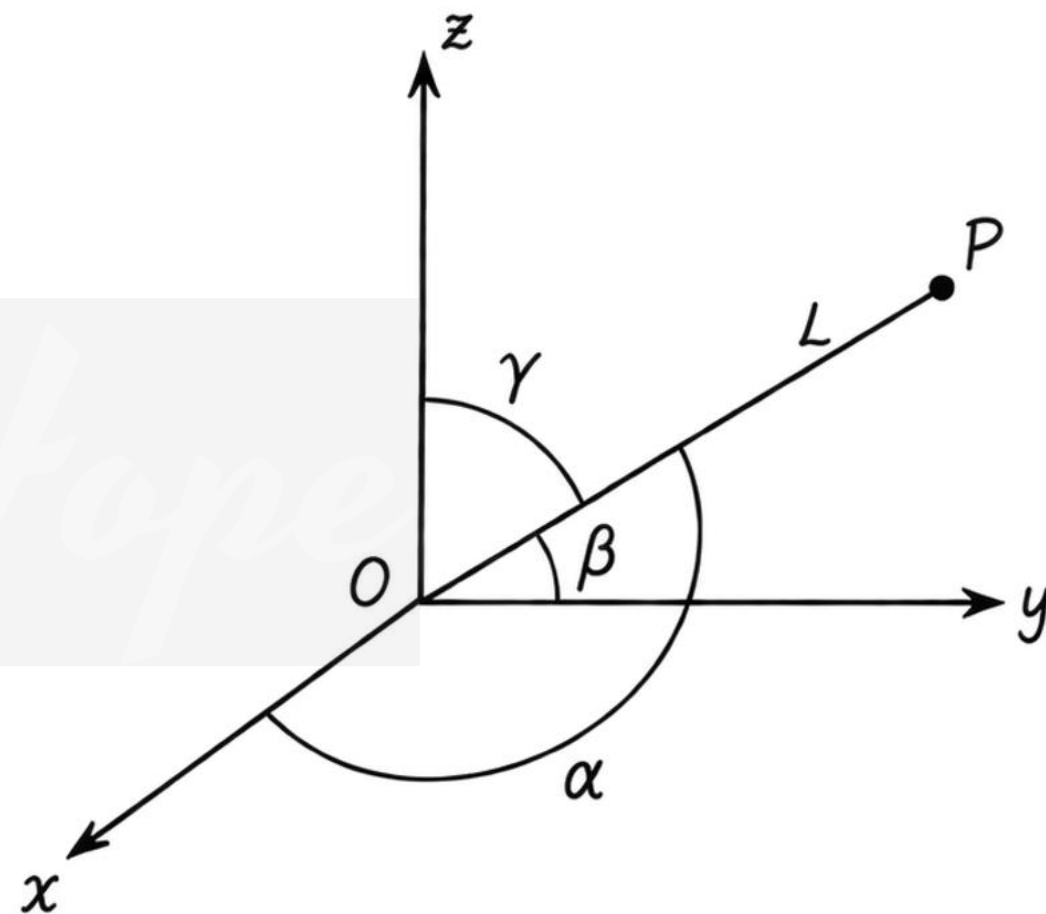
$$l^2 + m^2 + n^2 = 1$$

A directed line has one set of direction cosines. Reversing its direction reverses all three signs.

$$(l, m, n) \rightarrow (-l, -m, -n)$$

$$\alpha \rightarrow \pi - \alpha, \quad \beta \rightarrow \pi - \beta, \quad \gamma \rightarrow \pi - \gamma$$

For a line away from the origin, use a parallel line through the origin with the same orientation.



$$\text{d.c.} = (l, m, n)$$

DIRECTION RATIOS AND NORMALISATION

Direction ratios

Direction ratios are any three numbers proportional to the direction cosines. They are also called direction numbers.

$$a = \lambda l, \quad b = \lambda m, \quad c = \lambda n$$

$$\lambda \neq 0$$

$$(a, b, c) \neq (0, 0, 0)$$

Multiplying all ratios by the same nonzero number gives the same line direction. There are infinitely many such sets.

$$(a, b, c) \text{ and } (ta, tb, tc), \quad t \neq 0$$

Convert to direction cosines

$$l = ka, \quad m = kb, \quad n = kc$$

$$l^2 + m^2 + n^2 = 1$$

$$k^2(a^2 + b^2 + c^2) = 1$$

$$k = \pm \frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

$$(l, m, n) = \pm \frac{(a, b, c)}{\sqrt{a^2 + b^2 + c^2}}$$

Use the same sign for all three entries.
The sign selects the orientation.

FINDING DIRECTION COSINES

From direction angles

Take the cosine of each given direction angle.

$$\alpha = 90^\circ, \quad \beta = 60^\circ, \quad \gamma = 30^\circ$$

$$l = \cos 90^\circ = 0$$

$$m = \cos 60^\circ = \frac{1}{2}$$

$$n = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$(l, m, n) = \left(0, \frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

$$0^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = 1$$

positive axis	direction angles	direction cosines
x	$(0^\circ, 90^\circ, 90^\circ)$	$(1, 0, 0)$
y	$(90^\circ, 0^\circ, 90^\circ)$	$(0, 1, 0)$
z	$(90^\circ, 90^\circ, 0^\circ)$	$(0, 0, 1)$

From direction ratios

$$(a, b, c) = (2, -1, -2)$$

$$\sqrt{a^2 + b^2 + c^2} = \sqrt{4 + 1 + 4} = 3$$

$$(l, m, n) = \left(\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}\right)$$

$$\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{2}{3}\right)^2 = 1$$

The opposite orientation has all signs reversed.

$$\left(-\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$$

DIRECTION OF A LINE JOINING TWO POINTS

$$P(x_1, y_1, z_1), Q(x_2, y_2, z_2), P \neq Q$$

$$\Delta x = x_2 - x_1$$

$$\Delta y = y_2 - y_1$$

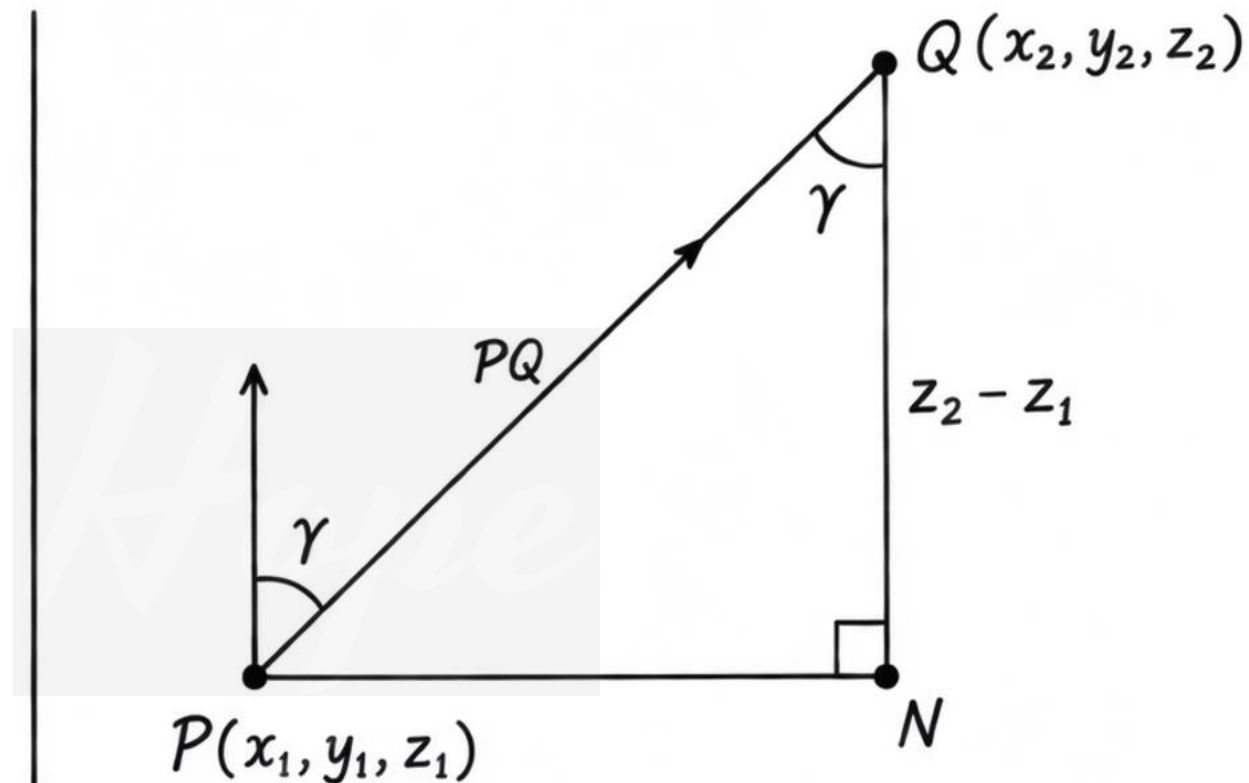
$$\Delta z = z_2 - z_1$$

$$PQ = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}$$

$$(l, m, n) = \frac{(\Delta x, \Delta y, \Delta z)}{PQ}$$

Coordinate differences give direction ratios.
Divide by the length to get direction cosines.

$$\cos \alpha = \frac{\Delta x}{PQ}, \quad \cos \beta = \frac{\Delta y}{PQ}, \quad \cos \gamma = \frac{\Delta z}{PQ}$$



Reversing the order of the points reverses the direction ratios.

$$(-\Delta x, -\Delta y, -\Delta z)$$



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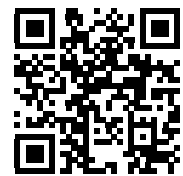


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