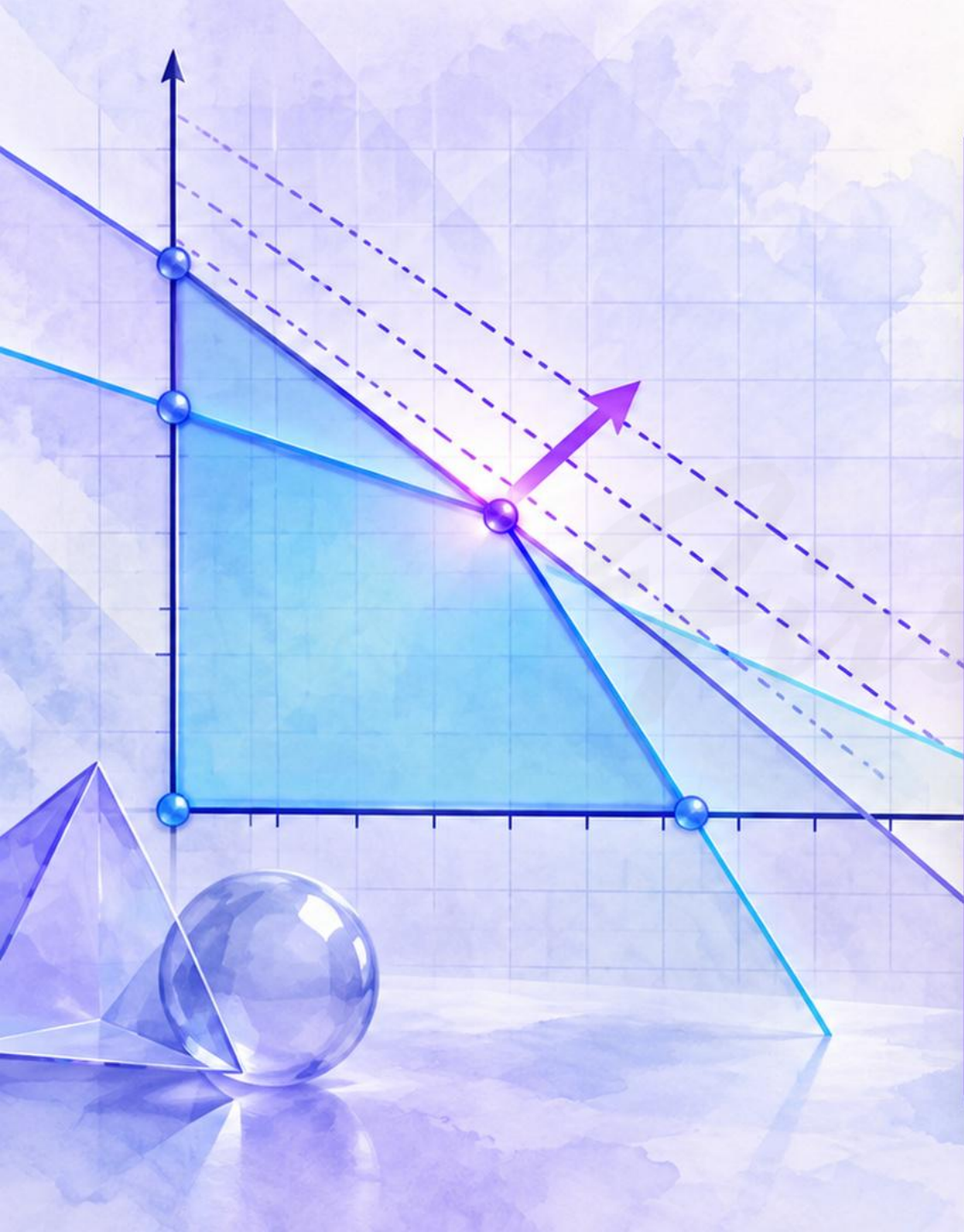




MATHEMATICS • CLASS 12

12

CHAPTER

Linear Programming

NCERT SOLUTIONS

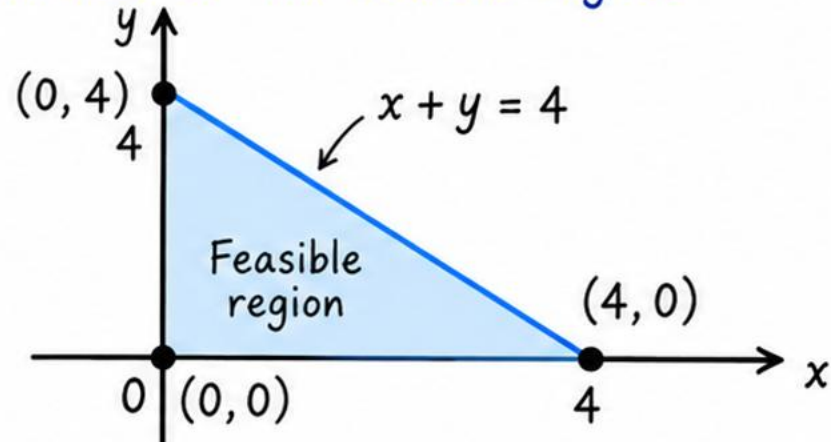
LINEAR PROGRAMMING – EXERCISE 12.1 Q1

Q. Maximise $Z = 3x + 4y$ subject to $x + y \leq 4$, $x \geq 0$, $y \geq 0$. Solve graphically.

Answer:

Find the feasible triangle

Consider the constraint $x + y \leq 4$ along with $x \geq 0$, $y \geq 0$. The line $x + y = 4$ cuts the axes at $(4, 0)$ and $(0, 4)$. With $x, y \geq 0$ and $x + y \leq 4$, the feasible region is the triangular region in the first quadrant bounded by the axes and the line $x + y = 4$. Hence, the feasible vertices are $(0, 0)$, $(4, 0)$ and $(0, 4)$. A linear objective function attains its maximum value at one of the vertices of the feasible region.



Compare the objective values

Evaluate the objective function $Z = 3x + 4y$ at each vertex of the feasible region.

Vertex (x, y)	$Z = 3x + 4y$
$(0, 0)$	$Z(0, 0) = 3(0) + 4(0) = 0$
$(4, 0)$	$Z(4, 0) = 3(4) + 4(0) = 12$
$(0, 4)$	$Z(0, 4) = 3(0) + 4(4) = 16$

The largest value of Z is 16, which occurs at the vertex $(0, 4)$. Therefore, the maximum value of the objective function is 16 at $(0, 4)$.

Hence, maximum $Z = 16$ at $(0, 4)$.

MINIMUM AT A VERTEX — EXERCISE 12.1 Q2

Q. Minimise $Z = -3x + 4y$ subject to $x + 2y \leq 8$, $3x + 2y \leq 12$, $x \geq 0$, $y \geq 0$.

Answer:

Determine the feasible vertices

The feasible region is in the first quadrant ($x \geq 0$, $y \geq 0$) and below the lines $x + 2y = 8$ and $3x + 2y = 12$.

The corner points (vertices) are obtained from the intersections of the boundary lines and the axes.

Intersection of the two lines:

$$\left. \begin{array}{l} x + 2y = 8 \\ 3x + 2y = 12 \end{array} \right\} \Rightarrow 2x = 4 \Rightarrow x = 2$$

Substitute in $x + 2y = 8$:

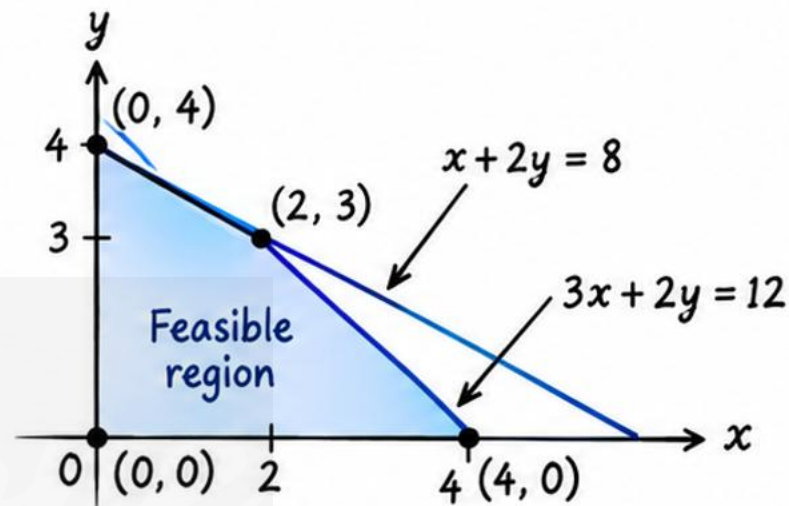
$$2 + 2y = 8 \Rightarrow 2y = 6 \Rightarrow y = 3.$$

Intersections with the axes:

On x -axis ($y = 0$): from $x + 2y = 8$, $x = 8$ (not feasible since $3x \leq 12$ gives $x \leq 4$), and from $3x + 2y = 12$, $x = 4$. Hence vertex $(4, 0)$.

On y -axis ($x = 0$): from $x + 2y = 8$, $y = 4$ (feasible), and from $3x + 2y = 12$, $y = 6$ (not feasible since $y \leq 4$).

Thus, the first-quadrant feasible vertices are $(0, 0)$, $(4, 0)$, $(2, 3)$ and $(0, 4)$.



Evaluate Z at each vertex

Substitute the coordinates of each vertex in $Z = -3x + 4y$.

Vertex (x, y)	$Z = -3x + 4y$
$(0, 0)$	$Z(0, 0) = -3(0) + 4(0) = 0$
$(4, 0)$	$Z(4, 0) = -3(4) + 4(0) = -12$
$(2, 3)$	$Z(2, 3) = -3(2) + 4(3) = -6 + 12 = 6$
$(0, 4)$	$Z(0, 4) = -3(0) + 4(4) = 16$

The least value is -12 , which occurs at $(4, 0)$.

Hence, minimum $Z = -12$ at $(4, 0)$.

INTERSECTING CONSTRAINTS — EXERCISE 12.1 Q3

Q. Maximise $Z = 5x + 3y$ subject to $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$.

Answer:

Find the feasible corners

The feasible region is the common region satisfying $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$.

The axes give $(0, 0)$, $(2, 0)$, and $(0, 3)$.

Find the intersection of $3x + 5y = 15$ and $5x + 2y = 10$.

$$3x + 5y = 15 \quad \dots(1)$$

$$5x + 2y = 10 \quad \dots(2)$$

Multiply (1) by 2 and (2) by 5:

$$6x + 10y = 30$$

$$\underline{-25x + 10y = 50}$$

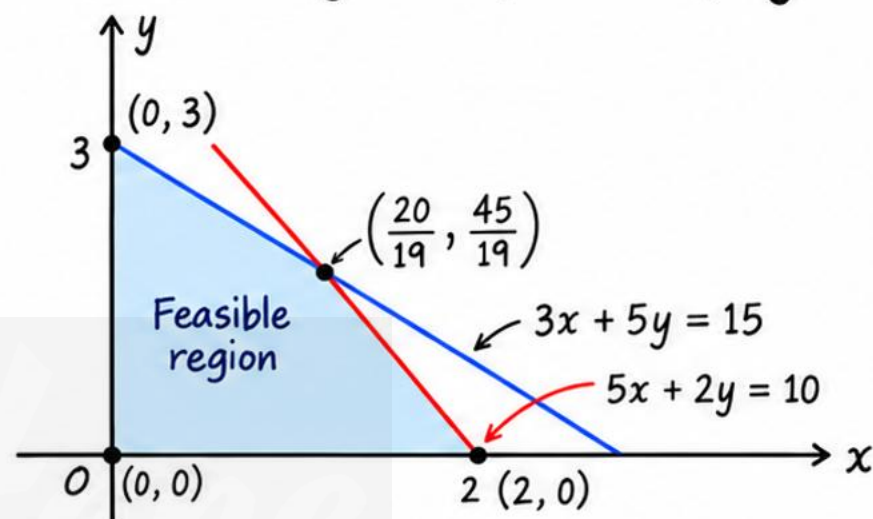
$$-19x = -20$$

$$x = \frac{20}{19}$$

$$\text{From (2), } 5\left(\frac{20}{19}\right) + 2y = 10 \Rightarrow 2y = 10 - \frac{100}{19} = \frac{90}{19}$$

$$y = \frac{45}{19}$$

So, the feasible corners are $(0, 0)$, $(2, 0)$, $(0, 3)$ and $(\frac{20}{19}, \frac{45}{19})$, which also lies in the feasible region.



Test the objective

Evaluate $Z = 5x + 3y$ at each feasible corner.

Corner (x, y)	$Z = 5x + 3y$
$(0, 0)$	$Z(0, 0) = 5(0) + 3(0) = 0$
$(2, 0)$	$Z(2, 0) = 5(2) + 3(0) = 10$
$(0, 3)$	$Z(0, 3) = 5(0) + 3(3) = 9$
$(\frac{20}{19}, \frac{45}{19})$	$Z(\frac{20}{19}, \frac{45}{19}) = 5(\frac{20}{19}) + 3(\frac{45}{19}) = \frac{100}{19} + \frac{135}{19} = \frac{235}{19}$

Since $\frac{235}{19}$ exceeds both 10 and 9, it is the maximum.

Hence, maximum $Z = \frac{235}{19}$ at $(\frac{20}{19}, \frac{45}{19})$.

MINIMUM ON AN UNBOUNDED REGION — EXERCISE 12.1 Q4

Q. Minimise $Z = 3x + 5y$ subject to $x + 3y \geq 3$, $x + y \geq 2$, $x \geq 0$, $y \geq 0$.

Answer:

Identify the lower boundary

The feasible region lies above both lines in the first quadrant. Their intersection satisfies $x + 3y = 3$ and $x + y = 2$; subtracting gives $y = \frac{1}{2}$ and $x = \frac{3}{2}$.

The relevant corners are $(0, 2)$, $(\frac{3}{2}, \frac{1}{2})$, and $(3, 0)$.

Solve the intersection:

$$\left. \begin{array}{l} x + 3y = 3 \\ x + y = 2 \end{array} \right\} \Rightarrow 2y = 1 \Rightarrow y = \frac{1}{2}.$$

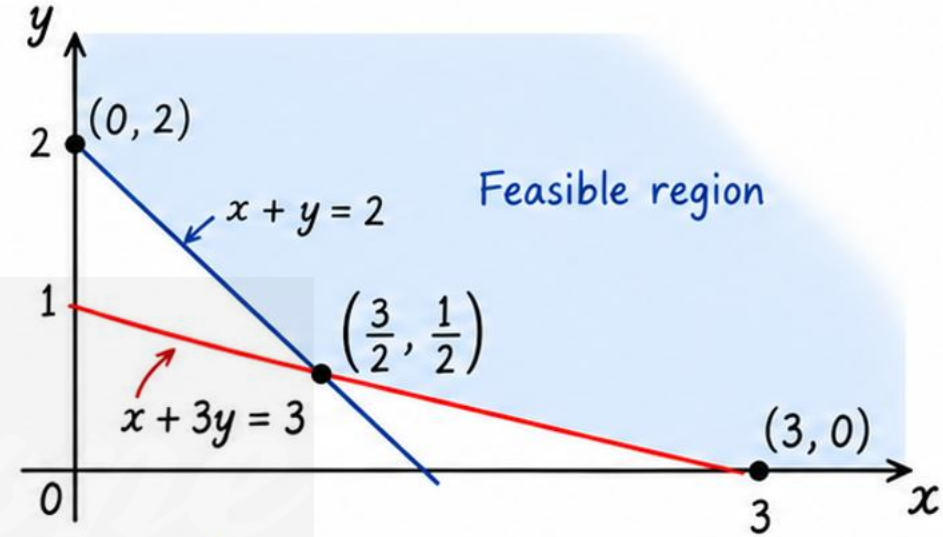
Substitute in $x + y = 2$: $x = 2 - \frac{1}{2} = \frac{3}{2}$.

The corner points are:

$(0, 2)$: $x = 0$ in $x + y = 2$ gives $y = 2$ (satisfies $x + 3y \geq 3$).

$(3, 0)$: $y = 0$ in $x + 3y = 3$ gives $x = 3$ (satisfies $x + y \geq 2$).

$(\frac{3}{2}, \frac{1}{2})$: intersection of the two lines.



Compare corner values

Evaluate $Z = 3x + 5y$ at each corner point.

$$Z(0, 2) = 3(0) + 5(2) = 10.$$

$$Z\left(\frac{3}{2}, \frac{1}{2}\right) = 3\left(\frac{3}{2}\right) + 5\left(\frac{1}{2}\right) = \frac{9}{2} + \frac{5}{2} = 7.$$

$$Z(3, 0) = 3(3) + 5(0) = 9.$$

The region is unbounded, but $Z = 3x + 5y$ increases as x or y grows, so no distant point can lower the value below the lower boundary minimum.

Hence, minimum $Z = 7$ at $(\frac{3}{2}, \frac{1}{2})$.

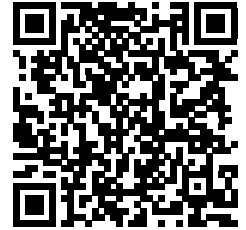


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