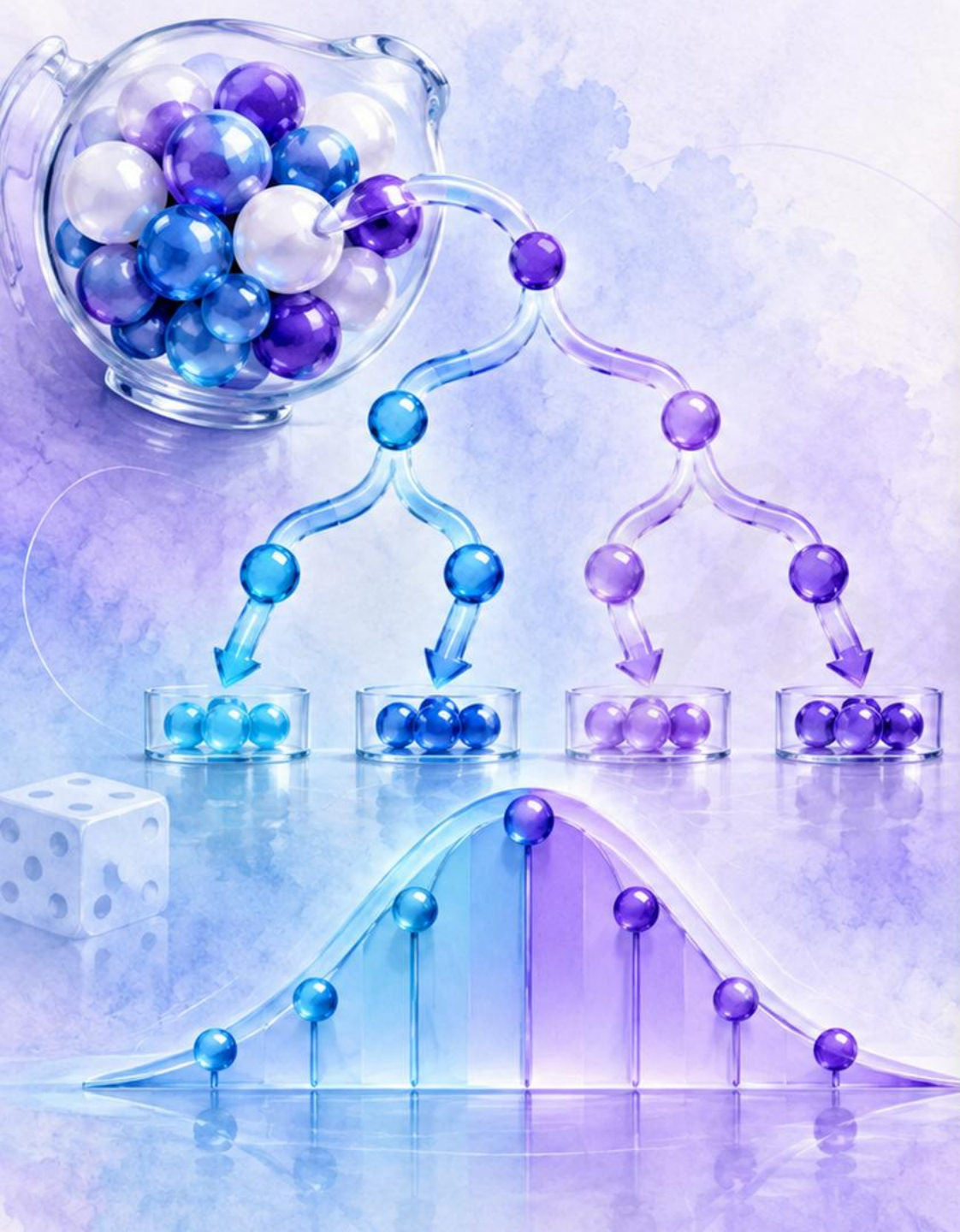




MATHEMATICS • CLASS 12

13

CHAPTER

Probability

CHAPTER NOTES

PROBABILITY: EVENTS AND NEW INFORMATION

Recall the basic ideas

Probability measures how likely an event is in a random experiment.

The sample space is the set of all possible outcomes. An event is a subset of the sample space.

S = sample space, $E \subseteq S$

$$0 \leq P(E) \leq 1, \quad P(S) = 1$$

For a finite sample space with equally likely outcomes, count favourable outcomes.

$$P(E) = \frac{n(E)}{n(S)}$$

Information can change probability

When we know that another event has occurred, only outcomes compatible with that information remain possible. This leads to conditional probability.

Most experiments in this chapter have equally likely outcomes, unless stated otherwise.

Addition rule recalled

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

The common outcomes must not be counted twice.

$$A \cap B = \emptyset \Rightarrow P(A \cup B) = P(A) + P(B)$$

CONDITIONAL PROBABILITY WITH THREE COINS

Toss three fair coins. Each ordered outcome has the same probability.

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

$$P(\text{each outcome}) = \frac{1}{8}$$

Before extra information

The first event is at least two heads. The second event is a tail on the first coin.

$$E = \{HHH, HHT, HTH, THH\}$$

$$F = \{THH, THT, TTH, TTT\}$$

$$E \cap F = \{THH\}$$

$$P(E) = \frac{4}{8} = \frac{1}{2}$$

$$P(F) = \frac{4}{8} = \frac{1}{2}$$

$$P(E \cap F) = \frac{1}{8}$$

Given a tail on the first coin

The reduced sample space has four outcomes. Only one also gives at least two heads.

$$F = \{THH, THT, TTH, TTT\}$$

$$P(E | F) = \frac{1}{4}$$

Read the vertical bar as 'given that'. The new information changes the probability.

$$P(E) = \frac{1}{2}, \quad P(E | F) = \frac{1}{4}$$

THE CONDITIONAL PROBABILITY FORMULA

Definition

Conditional probability is the probability of an event when another event is known to have occurred. Both events belong to the same sample space.

$$P(E | F) = \frac{P(E \cap F)}{P(F)}$$

$$P(F) > 0$$

The denominator is the probability of the event that is already known. The numerator is the probability that both events occur.

This formula is not defined when the conditioning event has zero probability.

$$P(F) = 0 \Rightarrow P(E | F) \text{ is not defined}$$

Why counting gives this formula

For equally likely outcomes, use the reduced sample space.

$$\begin{aligned} P(E | F) &= \frac{n(E \cap F)}{n(F)} \\ &= \frac{n(E \cap F) / n(S)}{n(F) / n(S)} \\ &= \frac{P(E \cap F)}{P(F)} \end{aligned}$$

The probability formula also works when outcomes are not equally likely. In that case, use their actual probabilities rather than just counting outcomes.

CONDITIONAL PROBABILITY: BASIC PROPERTIES

Assume that the conditioning event has positive probability. $P(F) > 0$

Certain events and the range

$$S \cap F = F, \quad F \cap F = F$$

$$P(S|F) = \frac{P(F)}{P(F)} = 1$$

$$P(F|F) = \frac{P(F)}{P(F)} = 1$$

An event that is already known has conditional probability one.

$$E \cap F \subseteq F$$

$$0 \leq P(E \cap F) \leq P(F)$$

$$0 \leq P(E|F) \leq 1$$

The complement rule

A prime means the complement: the event does not occur.

$$P(E'|F) = 1 - P(E|F)$$

Short proof

$$E \cup E' = S, \quad E \cap E' = \emptyset$$

$$\begin{aligned} P(E|F) + P(E'|F) &= P(S|F) \\ &= 1 \end{aligned}$$

$$P(E'|F) = 1 - P(E|F)$$

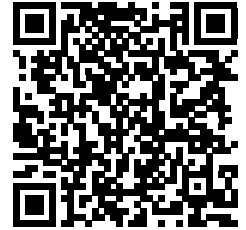


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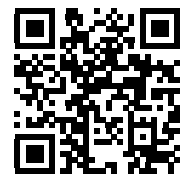


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