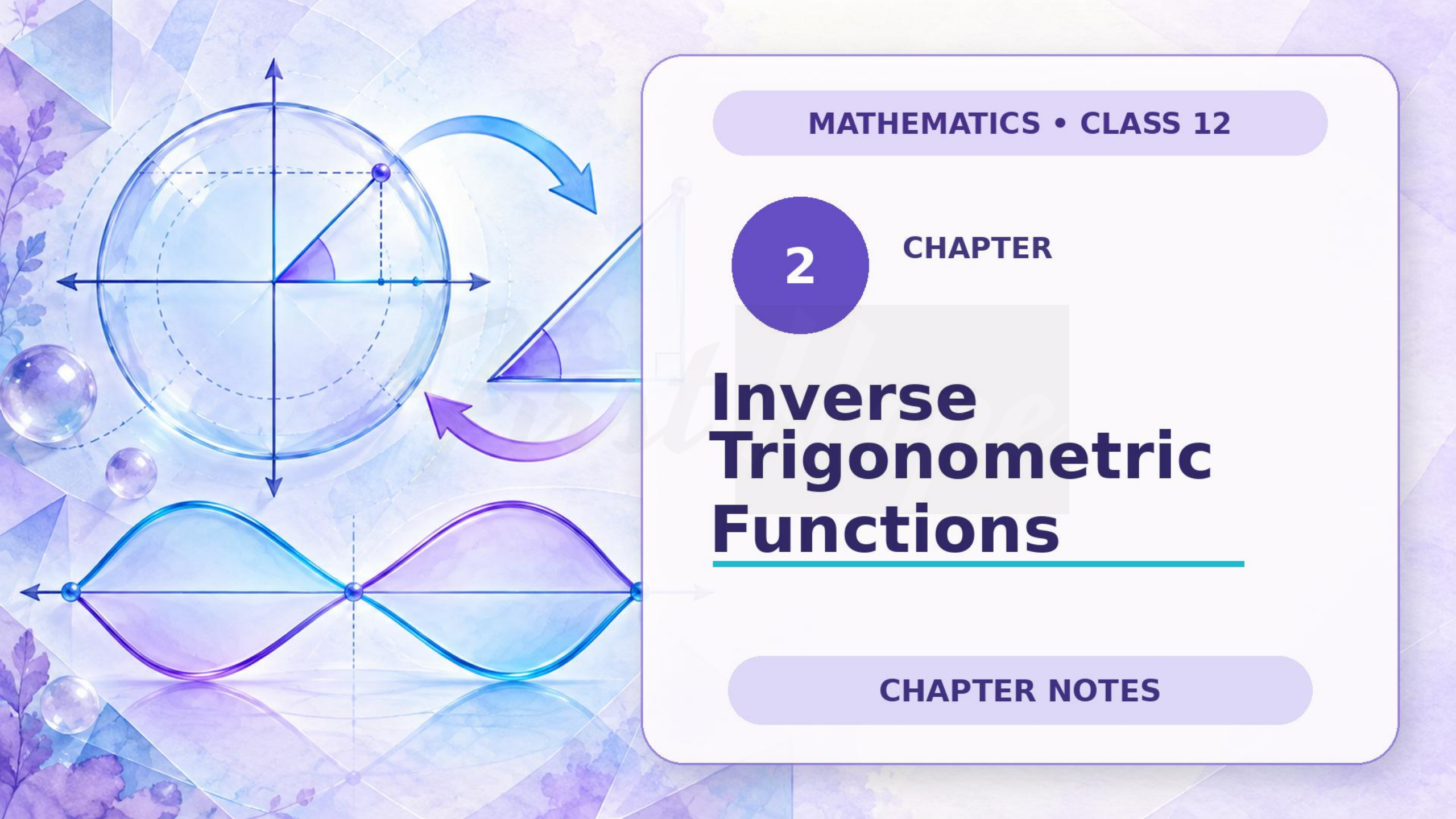




MATHEMATICS • CLASS 12

2

CHAPTER

Inverse Trigonometric Functions

CHAPTER NOTES

INVERSE FUNCTIONS & PRINCIPAL VALUES

Trigonometric functions repeat their values. On their full natural domains, they are not one-one, so we must restrict the domain before defining an inverse.

Reverse the function

$f: X \rightarrow Y$ is bijective; $y = f(x) \Leftrightarrow x = f^{-1}(y)$.

$f^{-1}(f(x)) = x, x \in X; f(f^{-1}(y)) = y, y \in Y$.

The inverse swaps the domain and range.
Different suitable restricted intervals give different branches.

Important notation

$\sin^{-1} x \neq (\sin x)^{-1}; (\sin x)^{-1} = \frac{1}{\sin x},$ wherever defined.

The superscript means inverse function, not reciprocal. The same caution applies to all six inverse functions. Inverse trigonometric functions are useful in calculus, science and engineering.

Principal value branch

We choose one standard branch for each inverse function. Its output is the principal value. Unless a branch is stated, use this principal branch.

An inverse trigonometric function takes a ratio as input and returns an angle.

NATURAL TRIGONOMETRIC DOMAINS

Function	Natural domain	Range
$\sin x$	$\mathbb{R}$	$[-1, 1]$
$\cos x$	$\mathbb{R}$	$[-1, 1]$
$\tan x$	$\mathbb{R} \setminus \left\{ \frac{(2n+1)\pi}{2} : n \in \mathbb{Z} \right\}$	$\mathbb{R}$
$\cot x$	$\mathbb{R} \setminus \{ n\pi : n \in \mathbb{Z} \}$	$\mathbb{R}$
$\sec x$	$\mathbb{R} \setminus \left\{ \frac{(2n+1)\pi}{2} : n \in \mathbb{Z} \right\}$	$(-\infty, -1] \cup [1, \infty)$
$\operatorname{cosec} x$	$\mathbb{R} \setminus \{ n\pi : n \in \mathbb{Z} \}$	$(-\infty, -1] \cup [1, \infty)$

- Tangent and secant are undefined at odd multiples of a right angle.
- Cotangent and cosecant are undefined at integer multiples of a straight angle.
- $\sec x = \frac{1}{\cos x}$; $\operatorname{cosec} x = \frac{1}{\sin x}$. $\mathbb{R} \setminus (-1, 1) = (-\infty, -1] \cup [1, \infty)$.
- The endpoints of the reciprocal ranges are included.
- Natural-domain functions must be restricted to suitable intervals to make them bijective.

INVERSE SINE & ITS GRAPH

$$\sin^{-1} : [-1, 1] \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right].$$

Arc sine gives the unique angle in the principal range whose sine is the input.

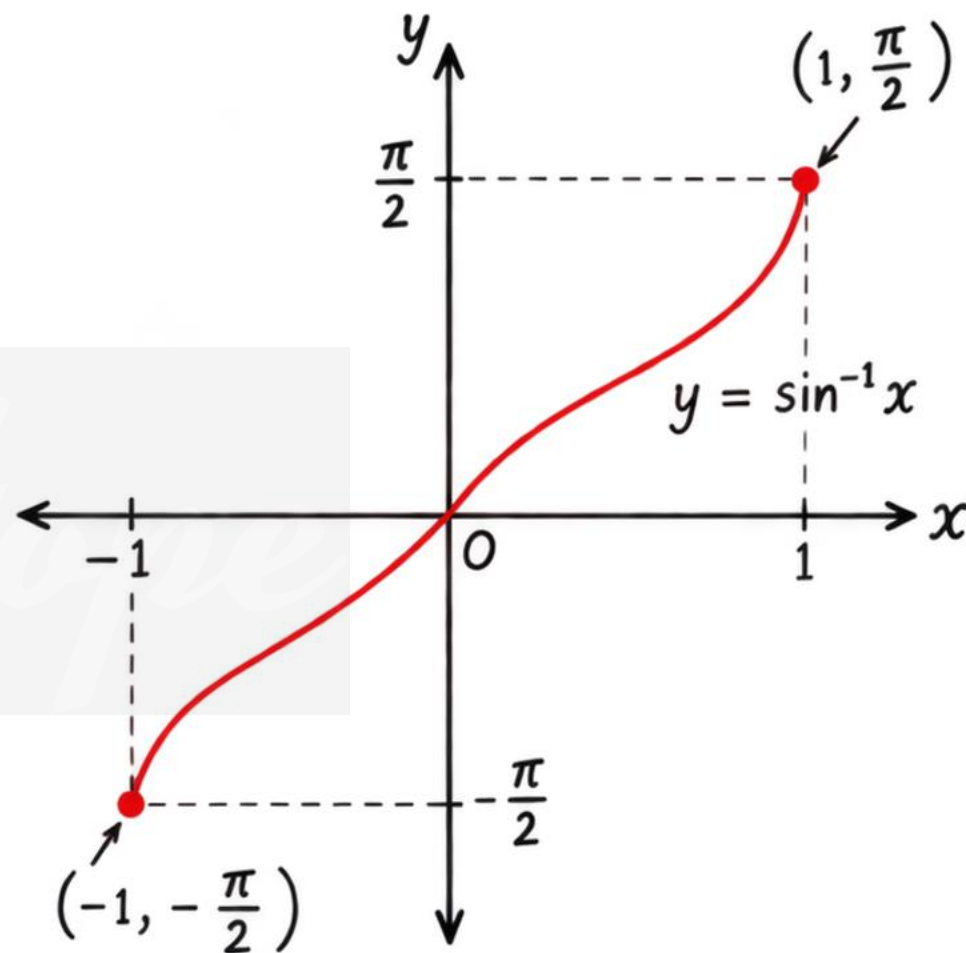
$$y = \sin^{-1} x \iff \sin y = x, \quad -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}.$$

$$\sin(\sin^{-1} x) = x, \quad -1 \leq x \leq 1;$$

$$\sin^{-1}(\sin \theta) = \theta, \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}.$$

Restrict sine to the principal interval to make it bijective.

Other branches: $\left[-\frac{3\pi}{2}, -\frac{\pi}{2}\right]$ and $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ are other possible inverse ranges.



Inverse graphs: Inverse graphs reflect the restricted original graph in the line of equality.

$y = x$; $(a, b) \leftrightarrow (b, a)$. Swap the coordinates of corresponding points.

INVERSE COSINE & ITS GRAPH

$$\cos^{-1} : [-1, 1] \rightarrow [0, \pi].$$

Arc cosine gives the unique angle in the principal range whose cosine is the input.

$$y = \cos^{-1} x \Leftrightarrow \cos y = x, \quad 0 \leq y \leq \pi.$$

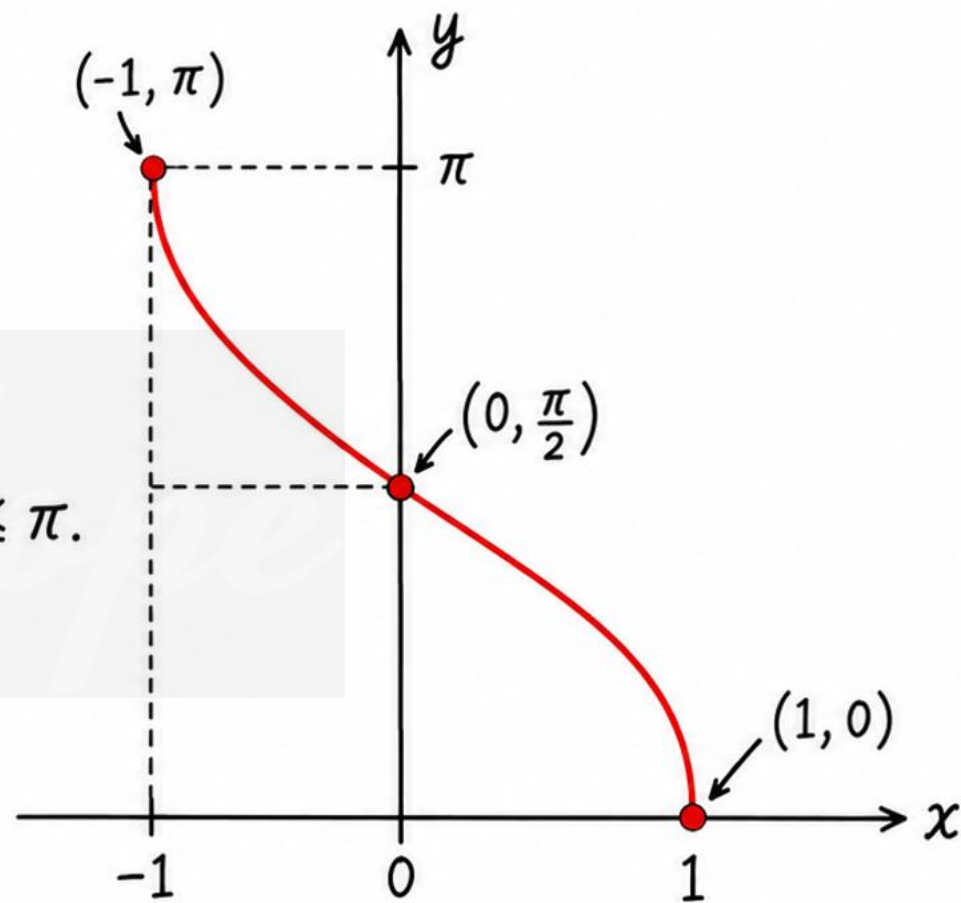
$$\cos(\cos^{-1} x) = x, \quad -1 \leq x \leq 1; \quad \cos^{-1}(\cos \theta) = \theta, \quad 0 \leq \theta \leq \pi.$$

Restrict cosine to the principal interval to make it bijective.

Other branches:

$[-\pi, 0]$ and $[\pi, 2\pi]$ are other possible inverse ranges.

Both the input interval and principal output interval include their endpoints.
The inverse graph is a reflection of the restricted cosine graph.



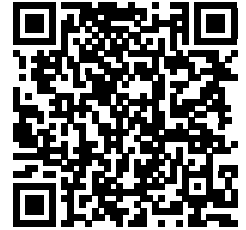


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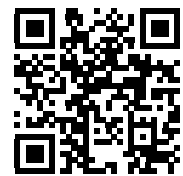


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