

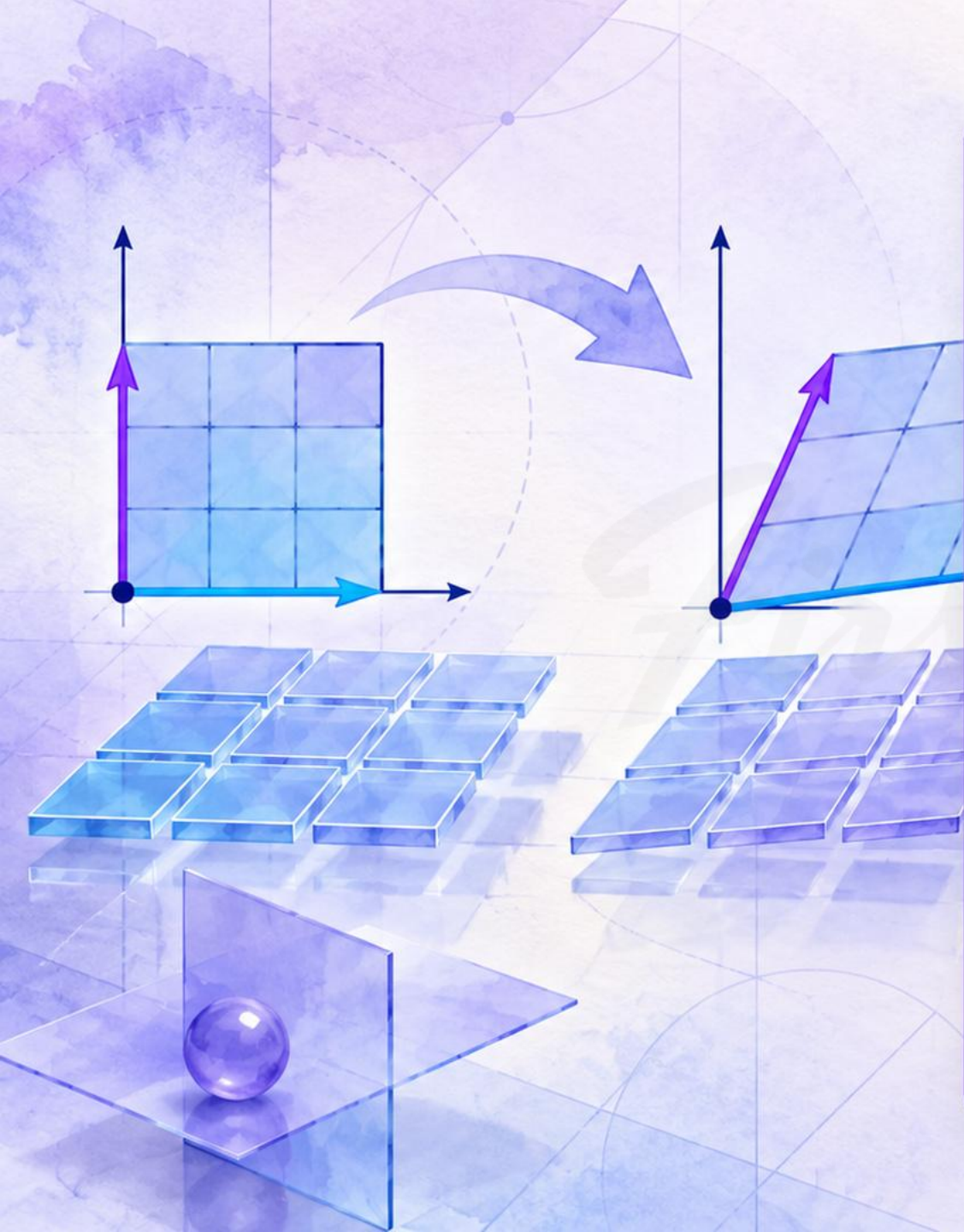
MATHEMATICS • CLASS 12

4

CHAPTER

Determinants

CHAPTER NOTES



DETERMINANTS: MEANING & SMALL ORDERS

A determinant assigns a number to a square matrix.
It helps us decide whether a square matrix has an inverse
and whether a linear system has a unique solution.

Smallest order

$$A = \begin{bmatrix} a \end{bmatrix} \Rightarrow |A| = a.$$

$$A = \begin{bmatrix} -5 \end{bmatrix} \Rightarrow |A| = -5.$$

A determinant can be negative, zero or positive.

Notation: $|A| = \det A = \Delta$.

These vertical bars mean determinant, not absolute value of the matrix.

Only square matrices have determinants.

Two rows and two columns

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow |A| = ad - bc.$$

Multiply the main-diagonal entries, then subtract the product of the other diagonal.

$$\begin{vmatrix} 2 & 4 \\ -1 & 2 \end{vmatrix} = 2 \times 2 - 4 \times (-1) = 8.$$

$$\begin{vmatrix} x & x+1 \\ x-1 & x \end{vmatrix} = x^2 - (x+1)(x-1) \\ = x^2 - (x^2 - 1) = 1.$$

Keep the subtraction sign and brackets around negative entries.

THIRD-ORDER DETERMINANTS

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Expand along the first row. For each entry, delete its row and column, evaluate the remaining determinant, and apply the sign.

First term

$$+a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} \\ = a_{11}(a_{22}a_{33} - a_{23}a_{32}).$$

Second term

$$-a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} \\ = -a_{12}(a_{21}a_{33} - a_{23}a_{31}).$$

Third term

$$+a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \\ = a_{13}(a_{21}a_{32} - a_{22}a_{31}).$$

Add these three signed terms.

$$\Delta = a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31}.$$

The first-row signs are plus, minus, plus.

CHOOSING A ROW OR COLUMN

A determinant can be expanded along any row or column. Every choice gives the same value. Choose a row or column with many zero entries to save calculation.

Along the second row

$$\Delta = -a_{21}(a_{12}a_{33} - a_{13}a_{32}) + a_{22}(a_{11}a_{33} - a_{13}a_{31}) - a_{23}(a_{11}a_{32} - a_{12}a_{31}).$$

Along the first column

$$\Delta = a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{21}(a_{12}a_{33} - a_{13}a_{32}) + a_{31}(a_{12}a_{23} - a_{13}a_{22}).$$

Sign pattern

$$= \begin{vmatrix} +, -, + \\ -, +, - \\ +, -, + \end{vmatrix}$$

sign at position (i,j) is $(-1)^{i+j}$.

Use the column with zeros

$$\Delta = \begin{vmatrix} 1 & 2 & 4 \\ -1 & 3 & 0 \\ 4 & 1 & 0 \end{vmatrix}$$

$$\Delta = 4 \begin{vmatrix} -1 & 3 \\ 4 & 1 \end{vmatrix} - 0 + 0 = 4[(-1)(1) - 3(4)] = 4(-13) = -52.$$

Only the nonzero entry contributes to this expansion.

SCALING & VARIABLE DETERMINANTS

Scaling every entry

If every entry of a square matrix is multiplied by the same scalar, the determinant is multiplied by that scalar raised to the order.

$$A = kB, \text{ order } n \Rightarrow |A| = k^n |B|.$$

For orders one, two and three, the scaling factors are:
 k, k^2, k^3 .

$$B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} \quad A = \begin{bmatrix} 2 & 2 \\ 4 & 0 \end{bmatrix} = 2B.$$

$$|B| = 1 \times 0 - 1 \times 2 = -2.$$

$$|A| = 2 \times 0 - 2 \times 4 = -8 = 2^2(-2).$$

Multiplying the matrix by a scalar does not usually multiply its determinant by only that scalar.

Find the unknown value

$$\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}.$$

Evaluate both determinants first.

$$3 - x^2 = 3 - 8.$$

$$x^2 = 8.$$

$$x = \pm 2\sqrt{2}.$$

Both signs are needed because either value has the same square.

The scaling power depends on the order of the square matrix.

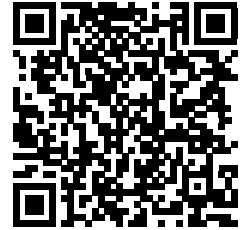


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