

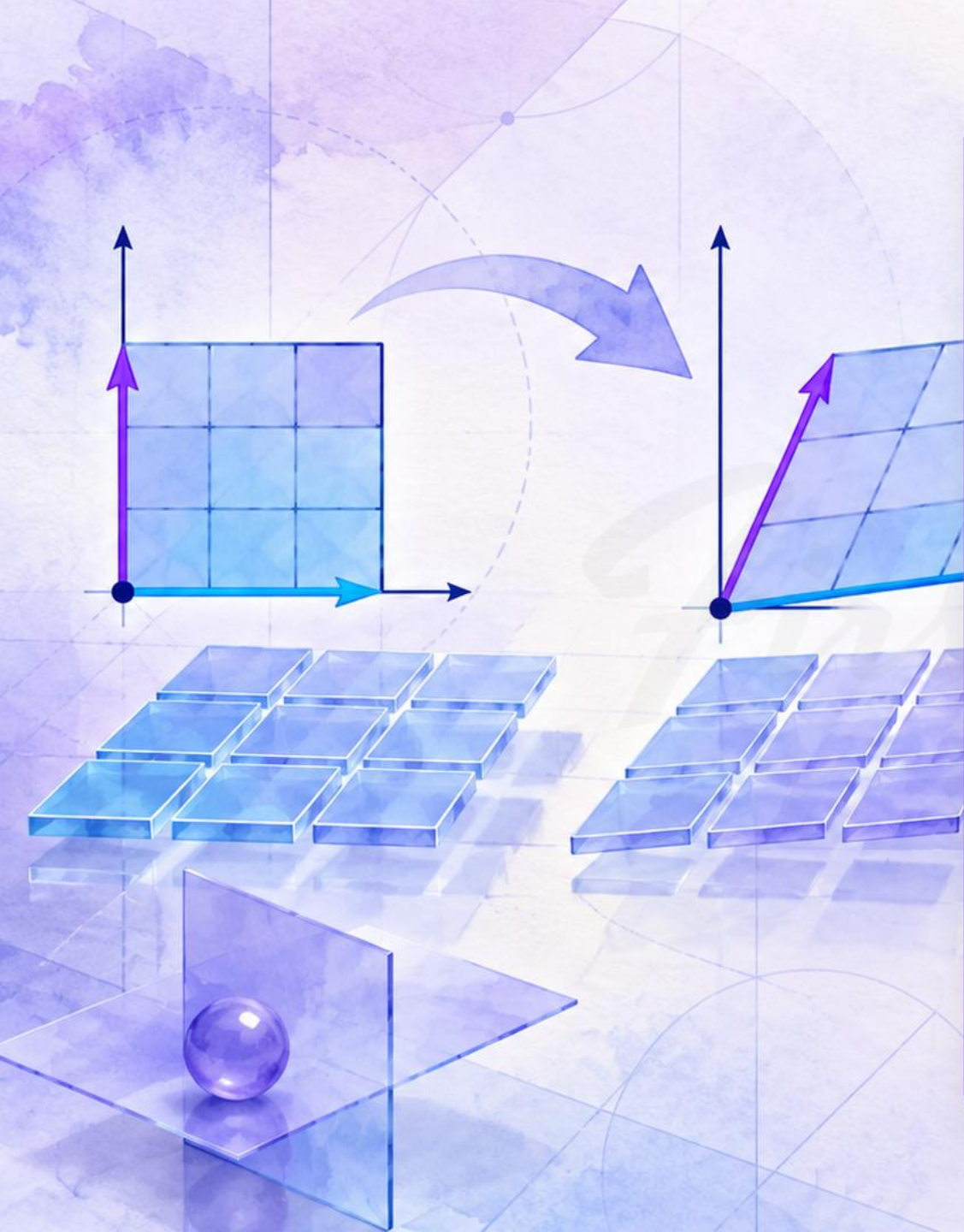
MATHEMATICS • CLASS 12

4

CHAPTER

Determinants

NCERT SOLUTIONS



DETERMINANT OF ORDER TWO - EXERCISE 4.1 Q1

Q. Evaluate the determinant $\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$.

Answer:

Rule: For a determinant of order two, multiply the entries on the main diagonal and subtract the product on the other diagonal.

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$$

Here, the main diagonal runs from the upper left to the lower right. Its product is

$$2 \times (-1) = -2.$$

The other diagonal runs from the upper right to the lower left. Its product is

$$4 \times (-5) = -20.$$

Therefore, using the rule, we get

$$\begin{aligned} \begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix} &= 2 \times (-1) - 4 \times (-5) \\ &= -2 - (-20) \\ &= -2 + 20 \\ &= 18. \end{aligned}$$

Subtracting a negative number is the same as adding the corresponding positive number.

Hence, $\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix} = 18.$

TRIGONOMETRIC DETERMINANT - EXERCISE 4.1 Q2(i)

Q. Evaluate the determinant $\begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix}$.

Answer:

Rule: For a determinant of order two, subtract the product on the other diagonal from the product on the main diagonal.

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$$

Applying this rule to the given determinant, we get

$$\begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix} = (\cos \theta)(\cos \theta) - (-\sin \theta)(\sin \theta).$$

The two diagonal products are

$$(\cos \theta)(\cos \theta) = \cos^2 \theta,$$

$$(-\sin \theta)(\sin \theta) = -\sin^2 \theta.$$

Subtracting the negative second product changes the subtraction into addition. Thus,

$$\begin{aligned} \begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix} &= \cos^2 \theta - (-\sin^2 \theta) \\ &= \cos^2 \theta + \sin^2 \theta. \end{aligned}$$

We now use the standard trigonometric identity

$$\sin^2 \theta + \cos^2 \theta = 1.$$

Hence, the determinant has the same value for every real angle.

$$\text{Hence, } \begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix} = 1.$$

ALGEBRAIC DETERMINANT - EXERCISE 4.1 Q2(ii)

Q. Evaluate the determinant $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$.

Answer:

Applying the rule

For a determinant of order two, we use

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$$

Therefore, the given determinant is

$$\Delta = (x^2 - x + 1)(x + 1) - (x - 1)(x + 1).$$

Both products contain the same factor. Taking this factor outside gives

$$\Delta = (x + 1)[(x^2 - x + 1) - (x - 1)].$$

The minus sign before the second bracket changes the sign of every term inside it.

$$\begin{aligned} & (x^2 - x + 1) - (x - 1) \\ &= x^2 - x + 1 - x + 1. \end{aligned}$$

Simplifying the expression

Combining like terms inside the bracket gives

$$x^2 - x + 1 - x + 1 = x^2 - 2x + 2.$$

Hence,

$$\Delta = (x + 1)(x^2 - 2x + 2).$$

Expanding the product gives an equivalent form:

$$\begin{aligned} \Delta &= x^3 - 2x^2 + 2x + x^2 - 2x + 2 \\ &= x^3 - x^2 + 2. \end{aligned}$$

The linear terms cancel, and the squared terms combine.
Both forms represent the same determinant.

$$\text{Hence, } \Delta = (x + 1)(x^2 - 2x + 2) = x^3 - x^2 + 2.$$

SCALING AN ORDER TWO MATRIX – EXERCISE 4.1 Q3

Q. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then show that $|2A| = 4|A|$.

Answer:

The original determinant

For a determinant of order two, use

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$$

Thus,

$$\begin{aligned} |A| &= \begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix} \\ &= 1 \times 2 - 2 \times 4 \\ &= 2 - 8 = -6. \end{aligned}$$

Multiplying this determinant value gives

$$4|A| = 4 \times (-6) = -24.$$

Both calculations give the same value. Scaling both rows contributes one factor from each row.

$$2 \times 2 = 2^2 = 4.$$

$$\text{Hence, } |2A| = -24 = 4|A|.$$

The scaled determinant

Scalar multiplication multiplies every entry of the matrix by the scalar. Therefore,

$$2A = 2 \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 8 & 4 \end{bmatrix}.$$

Its determinant is

$$\begin{aligned} |2A| &= \begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix} \\ &= 2 \times 4 - 4 \times 8 \\ &= 8 - 32 = -24. \end{aligned}$$



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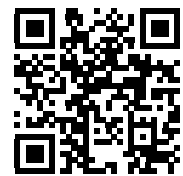


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