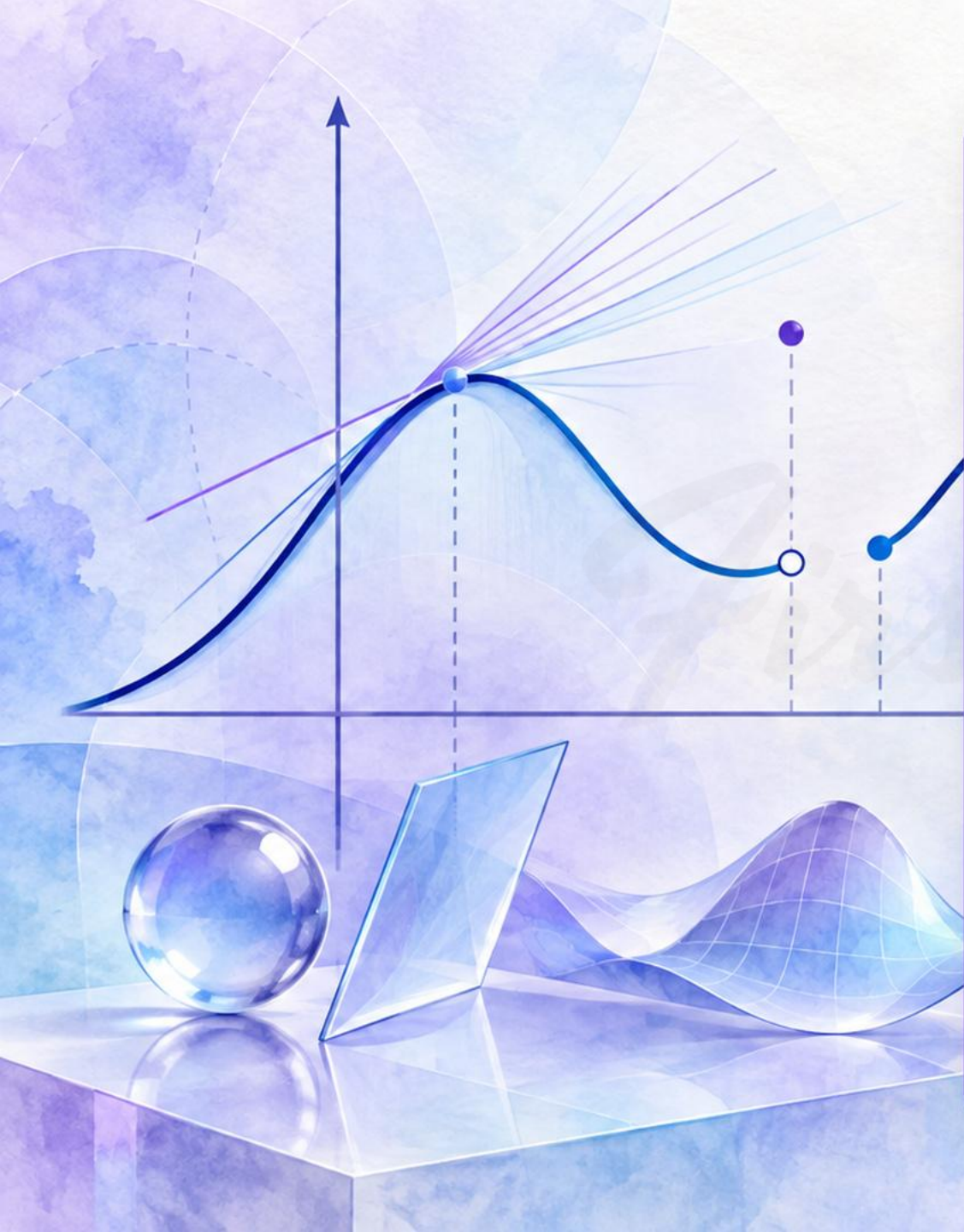


A complex mathematical illustration in shades of blue and purple. It features a 3D coordinate system with a wavy surface, a transparent sphere, a transparent cube, and a transparent plane. A 2D graph is overlaid on the 3D scene, showing a blue curve with a point where multiple tangent lines are drawn, illustrating the concept of differentiability. Other points on the curve are marked with solid and open circles, and vertical dashed lines connect them to the horizontal axis.

MATHEMATICS • CLASS 12

5

CHAPTER

Continuity and Differentiability

CHAPTER NOTES

CONTINUITY AT A POINT

A function is continuous at a point when its limit there equals its actual value.
The point must belong to the domain.

$$c \in \text{Domain}(f), \quad \lim_{x \rightarrow c} f(x) = f(c).$$

Check continuity

- Find the value at the point.
- Find the left-hand and right-hand limits.
- Check that both limits and the value agree.

$$\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x) = f(c).$$

At an interior point, all three quantities must exist and be equal, with a finite real value.

A simple illustration

$$f(x) = 2x + 3, \quad c = 1.$$

$$f(1) = 2(1) + 3 = 5.$$

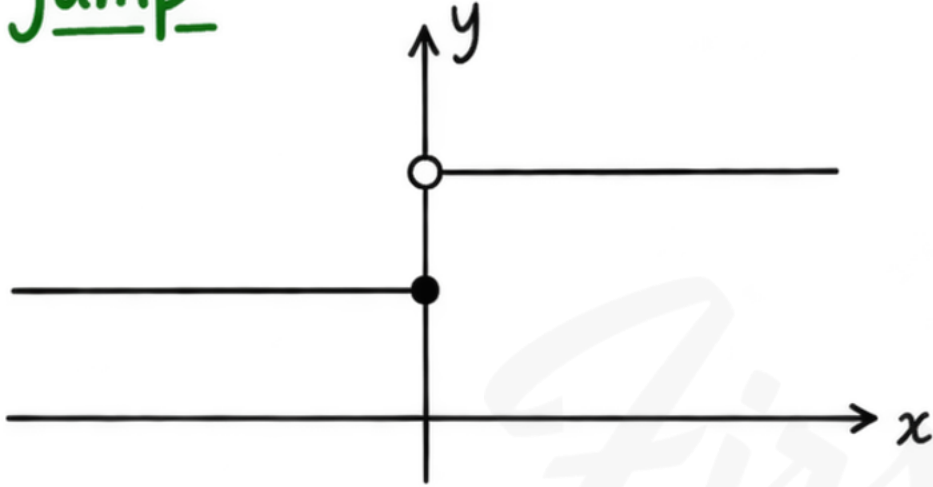
$$\lim_{x \rightarrow 1} f(x) = 2(1) + 3 = 5.$$

$$\lim_{x \rightarrow 1} f(x) = f(1).$$

So this function is continuous at the given point.

WHEN CONTINUITY FAILS

A jump

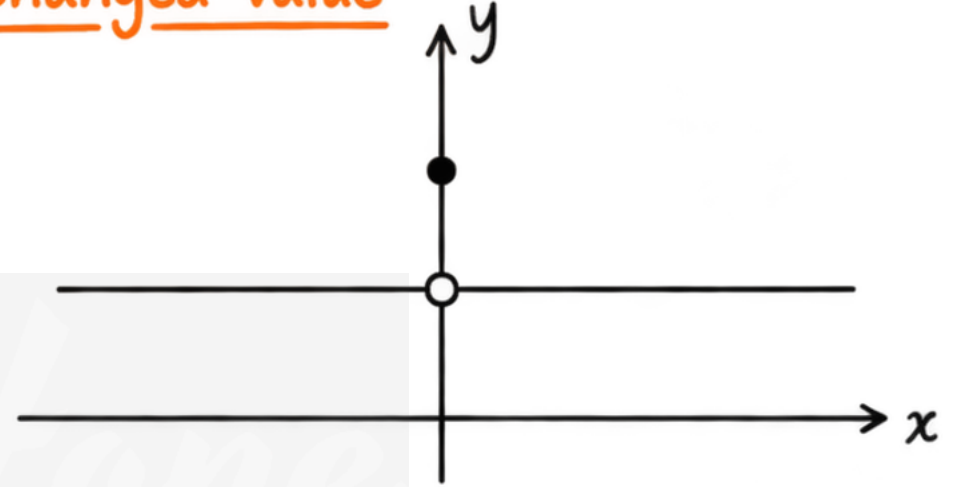


$$f(x) = \begin{cases} 1, & x \leq 0 \\ 2, & x > 0 \end{cases}$$

$$\text{LHL} = 1, \quad \text{RHL} = 2, \quad f(0) = 2$$

The one-sided limits differ, so the limit does not exist. The function is discontinuous here.

A changed value



$$g(x) = \begin{cases} 1, & x \neq 0 \\ 2, & x = 0 \end{cases}$$

$$\lim_{x \rightarrow 0} g(x) = 1 \neq g(0) = 2$$

The limit exists, but it does not equal the value. This function is also discontinuous here.

An open dot excludes a point; a filled dot includes it.

CONTINUITY: DOMAIN & ENDPOINTS

The whole domain

A function is continuous if it is continuous at every point of its domain.

Constant and identity functions are continuous on the real line.

$$f(x) = k : \lim_{x \rightarrow c} f(x) = k = f(c).$$

$$g(x) = x : \lim_{x \rightarrow c} g(x) = c = g(c).$$

Continuity is checked only where a function is defined.

$$h(x) = \frac{1}{x}, \quad \text{Domain}(h) = \mathbb{R} \setminus \{0\}.$$

The reciprocal function is continuous throughout its domain. It is undefined at zero, so zero is not a point at which we test its continuity.

A closed interval

$$f : [a, b] \rightarrow \mathbb{R}.$$

Use both one-sided limits at interior points.

At the left endpoint, approach from the right.

At the right endpoint, approach from the left.

$$\lim_{x \rightarrow a^+} f(x) = f(a).$$

$$\lim_{x \rightarrow b^-} f(x) = f(b).$$

Do not use values outside the stated domain.

THE MODULUS FUNCTION IS CONTINUOUS

$$f(x) = |x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

A negative point

$$c < 0: \lim_{x \rightarrow c} |x| = -c = |c|.$$

A positive point

$$c > 0: \lim_{x \rightarrow c} |x| = c = |c|.$$

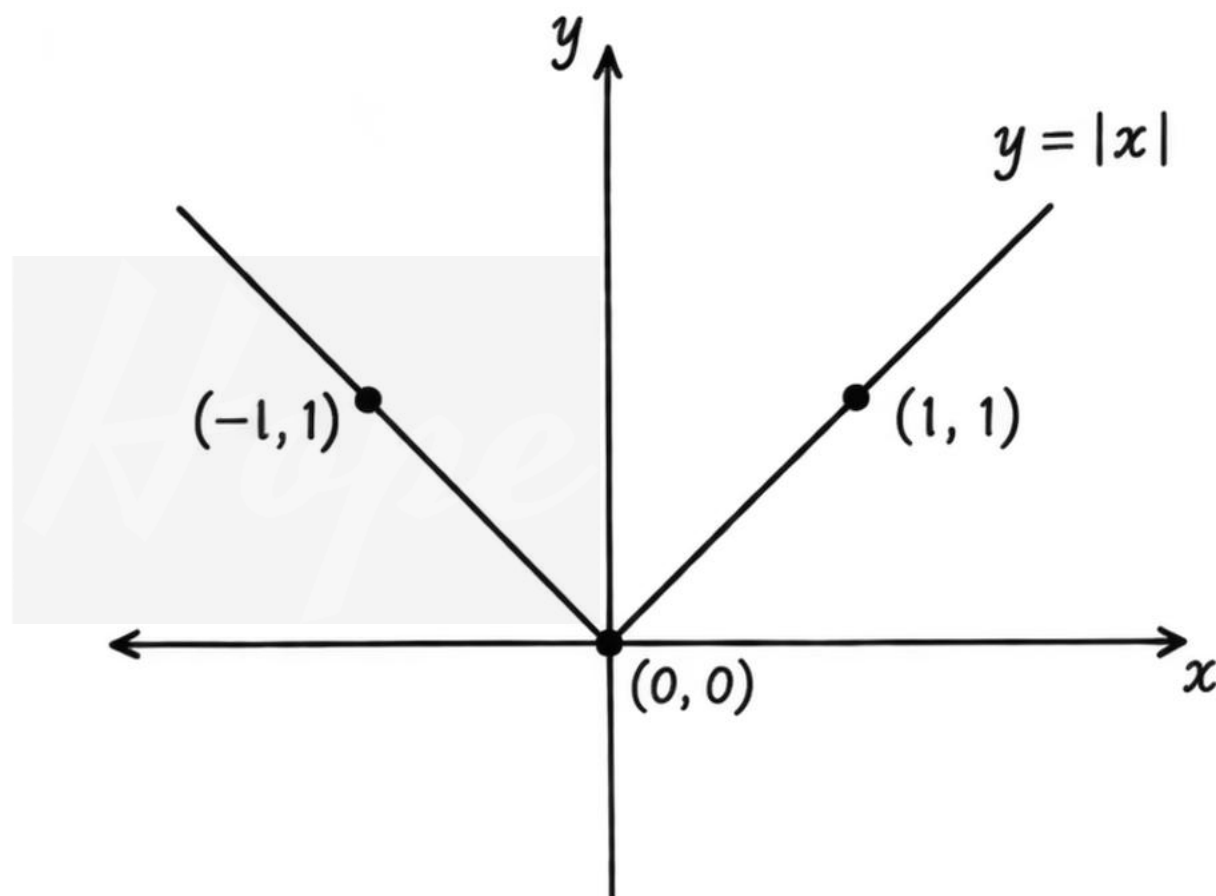
At the origin

$$\lim_{x \rightarrow 0^-} |x| = \lim_{x \rightarrow 0^-} (-x) = 0.$$

$$\lim_{x \rightarrow 0^+} |x| = \lim_{x \rightarrow 0^+} x = 0.$$

$$f(0) = 0.$$

Both limits equal the value at the origin. Hence the modulus function is continuous at every real point.





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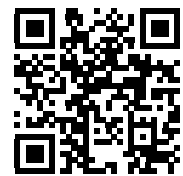


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