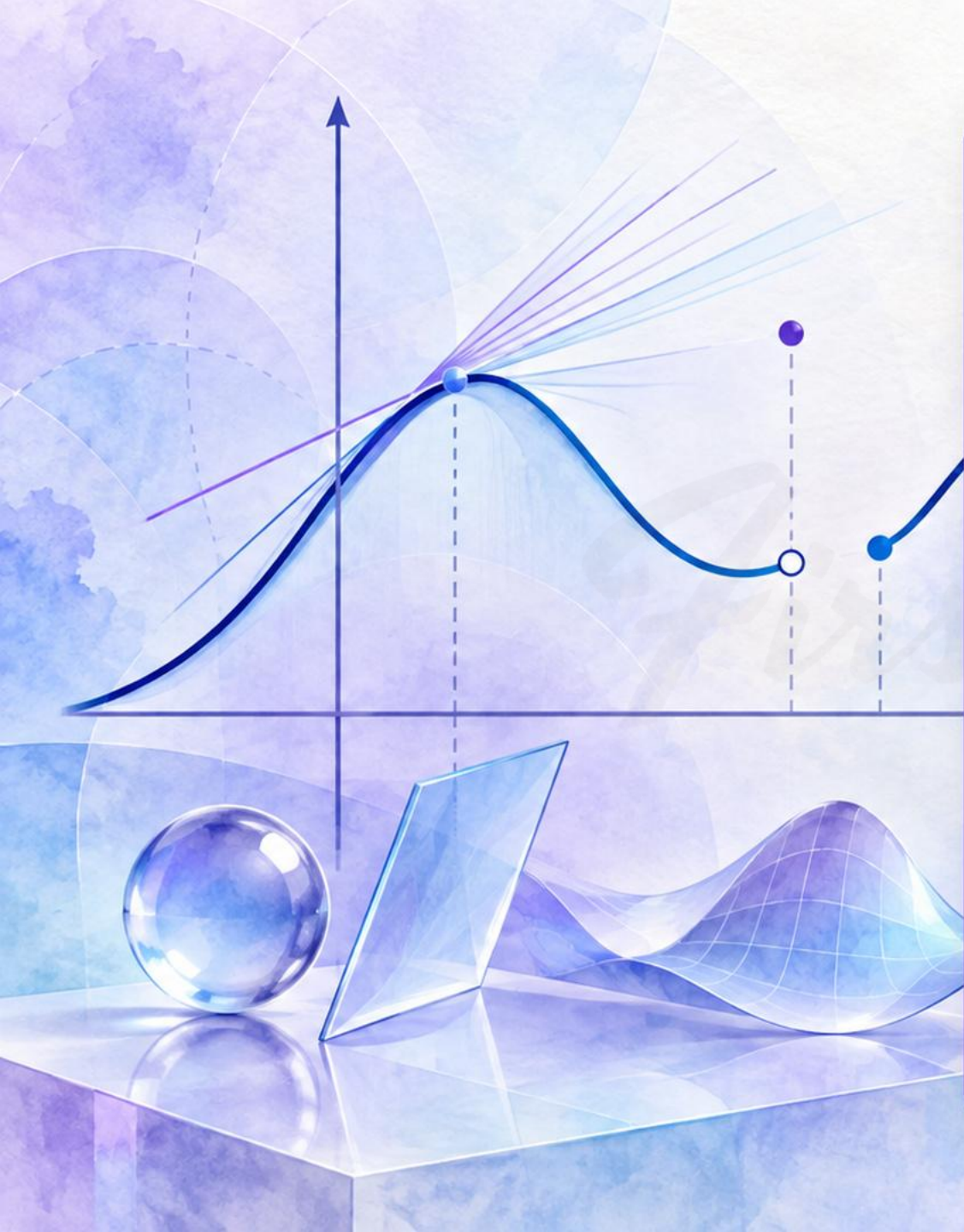




MATHEMATICS • CLASS 12

5

CHAPTER

Continuity and Differentiability

NCERT SOLUTIONS

A LINEAR FUNCTION IS CONTINUOUS – EXERCISE 5.1 Q1

Q. Prove $f(x) = 5x - 3$ is continuous at $x = 0, -3$ and 5 .

Answer:

A polynomial is continuous at every real number. In particular, for any c ,

$$\lim_{x \rightarrow c} (5x - 3) = 5c - 3 = f(c).$$

We now verify at each of the given points.

At $x = 0$:

$$\begin{array}{l} \lim_{x \rightarrow 0} (5x - 3) = 5(0) - 3 = -3. \\ f(0) = 5(0) - 3 = -3. \end{array} \quad \left. \vphantom{\begin{array}{l} \lim_{x \rightarrow 0} (5x - 3) = 5(0) - 3 = -3. \\ f(0) = 5(0) - 3 = -3. \end{array}} \right\} \begin{array}{l} \text{The limit equals the function value} \\ \text{at } x = 0, \text{ so } f \text{ is continuous at } 0. \end{array}$$

At $x = -3$:

$$\begin{array}{l} \lim_{x \rightarrow -3} (5x - 3) = 5(-3) - 3 = -18. \\ f(-3) = 5(-3) - 3 = -18. \end{array} \quad \left. \vphantom{\begin{array}{l} \lim_{x \rightarrow -3} (5x - 3) = 5(-3) - 3 = -18. \\ f(-3) = 5(-3) - 3 = -18. \end{array}} \right\} \begin{array}{l} \text{The limit equals the function value} \\ \text{at } x = -3, \text{ so } f \text{ is continuous at } -3. \end{array}$$

At $x = 5$:

$$\begin{array}{l} \lim_{x \rightarrow 5} (5x - 3) = 5(5) - 3 = 22. \\ f(5) = 5(5) - 3 = 22. \end{array} \quad \left. \vphantom{\begin{array}{l} \lim_{x \rightarrow 5} (5x - 3) = 5(5) - 3 = 22. \\ f(5) = 5(5) - 3 = 22. \end{array}} \right\} \begin{array}{l} \text{The limit equals the function value} \\ \text{at } x = 5, \text{ so } f \text{ is continuous at } 5. \end{array}$$

The equality of limit and function value holds at each of the requested points.

Hence, f is continuous at $0, -3$ and 5 .

CONTINUITY OF A QUADRATIC — EXERCISE 5.1 Q2

Q. Examine continuity of $f(x) = 2x^2 - 1$ at $x = 3$.

Answer:

Since f is a polynomial, it is continuous for all real numbers. In particular, its limit at 3 equals its value there.

First, find the value of the function at $x = 3$.

$$f(3) = 2(3)^2 - 1 = 18 - 1 = 17.$$

Next, evaluate the limit of $f(x)$ as x approaches 3. Using the limit law for polynomials (the limit equals the value obtained by direct substitution), we get

$$\lim_{x \rightarrow 3} (2x^2 - 1) = 2(3)^2 - 1 = 18 - 1 = 17.$$

Since the limit equals the function value at $x = 3$, i.e.,

$$\lim_{x \rightarrow 3} (2x^2 - 1) = 17 = f(3),$$

the continuity condition is satisfied.

Hence, f is continuous at $x = 3$.

A LINEAR FUNCTION – EXERCISE 5.1 Q3(a)

Q. Examine $f(x) = x - 5$ for continuity.

Answer:

For any real c , the function value is $f(c) = c - 5$.

The function $f(x) = x - 5$ is a linear (polynomial) function, and by the algebra of limits, the limit of a linear function is obtained by direct substitution. Hence,

$$\lim_{x \rightarrow c} (x - 5) = c - 5.$$

Thus $\lim_{x \rightarrow c} f(x) = c - 5 = f(c)$ for every $c \in \mathbb{R}$.

Since the limit at every point c exists and is equal to the function value at c , the function is continuous at each real number.

Hence, f is continuous for all real x .

A RATIONAL FUNCTION AND ITS DOMAIN – EXERCISE 5.1 Q3(b)

Q. Examine $f(x) = \frac{1}{x-5}$, $x \neq 5$, for continuity.

Answer:

The numerator and denominator are polynomials, so their quotient is continuous wherever the denominator is nonzero.

Hence, the function is defined and continuous for all x such that $x - 5 \neq 0$,
i.e. for $x \neq 5$.

Let $c \neq 5$ be any real number. Then, using the limit law for a rational function,

$$\lim_{x \rightarrow c} \frac{1}{x-5} = \frac{1}{c-5} = f(c).$$

Thus, f is continuous at every $c \neq 5$.

At $x = 5$, the formula $f(x) = \frac{1}{x-5}$ is undefined, so 5 is outside the stated domain.

Moreover, the values of $f(x)$ diverge as x approaches 5 from the left and from the right:

$$\lim_{x \rightarrow 5^-} \frac{1}{x-5} = -\infty, \quad \text{and} \quad \lim_{x \rightarrow 5^+} \frac{1}{x-5} = +\infty.$$

Since the one-sided limits are infinite, f is not continuous at $x = 5$, and no continuous extension exists there.

Hence, Continuous on $\mathbb{R} \setminus \{5\}$; undefined at 5.



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