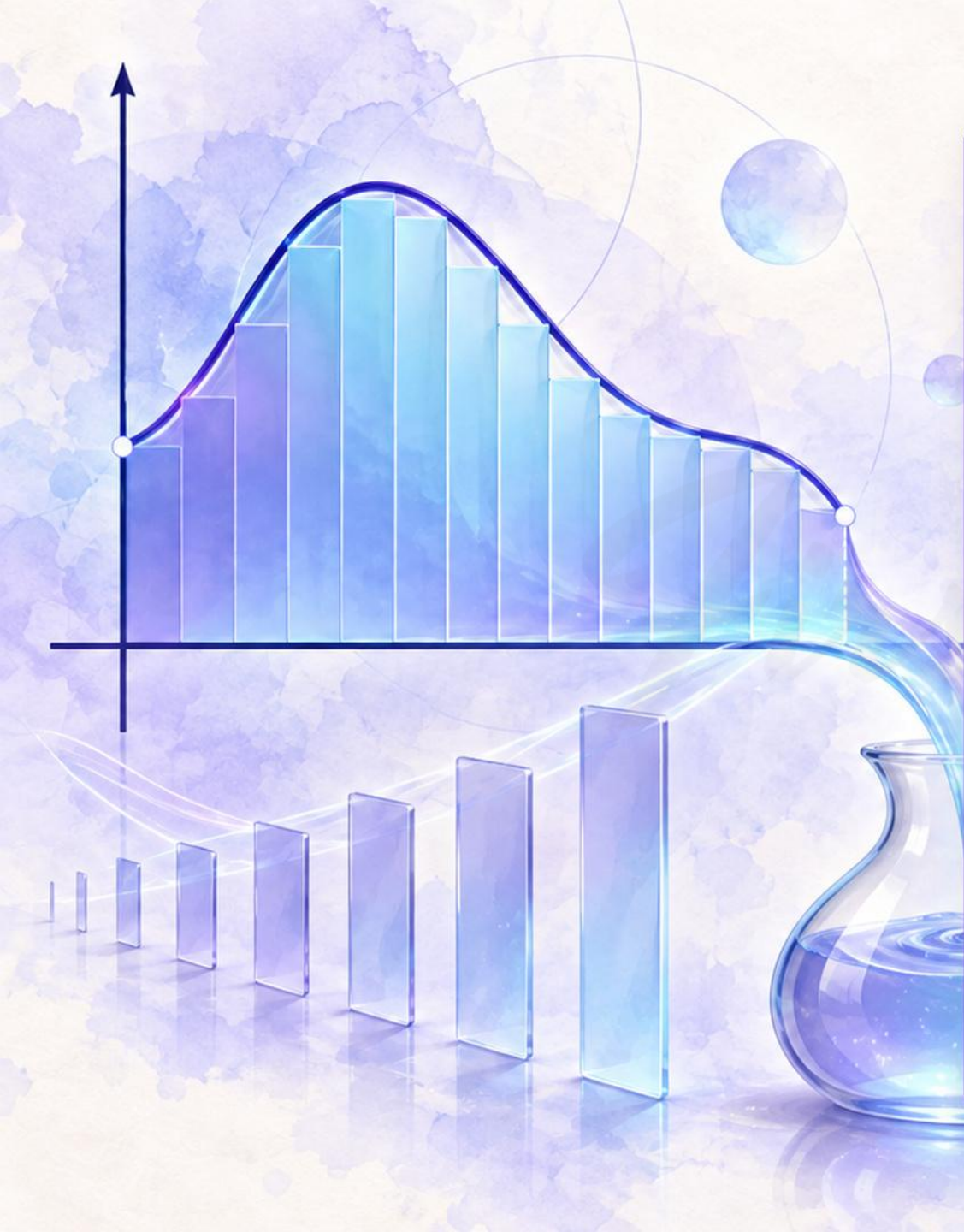




MATHEMATICS • CLASS 12

7

CHAPTER

Integrals

CHAPTER NOTES

INTEGRATION AND ANTIDERIVATIVES

Differentiation finds the rate of change of a known function.

Integration works in the opposite direction: it finds a function when its derivative is known.

An antiderivative of a function is a function whose derivative equals it.

$$F'(x) = f(x).$$

This is also called a primitive.

Integration helps find position from velocity and areas bounded by curves.

Indefinite integrals give a family of antiderivatives; definite integrals give a single value.

Reverse the derivative

$$\frac{d}{dx}(\sin x) = \cos x \Rightarrow \int \cos x \, dx = \sin x + C.$$

$$\frac{d}{dx}\left(\frac{x^3}{3}\right) = x^2 \Rightarrow \int x^2 \, dx = \frac{x^3}{3} + C.$$

$$\frac{d}{dx}(e^x) = e^x \Rightarrow \int e^x \, dx = e^x + C.$$

THE CONSTANT AND INTEGRAL NOTATION

Why a constant?

- The derivative of a constant is zero. So adding any constant gives another antiderivative.

$$\frac{d}{dx} (F(x) + C) = F'(x) = f(x), \quad C \in \mathbb{R}.$$

- On an interval, any two antiderivatives differ by a constant.

$$\begin{aligned} G'(x) = H'(x) = f(x) &\Rightarrow (G - H)' = 0 \\ &\Rightarrow G - H = C. \end{aligned}$$

Read the notation

$$\int f(x) dx = F(x) + C.$$

Integral sign:	$\int$
Integrand:	$f(x)$
Variable of integration:	x
Differential:	dx
Antiderivative:	$F(x)$
Constant of integration:	C

- Integration means finding the antiderivative.
An indefinite integral represents all such antiderivatives.

$$\int x^2 dx = \frac{x^3}{3} + C.$$

- A single primitive and the full family are different ideas.

POWER, LOG AND EXPONENTIAL INTEGRALS

Power rule

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1.$$

$$\int dx = x + C; \quad \int y^4 dy = \frac{y^5}{5} + C.$$

The integration variable need not always be the same letter.
Work where the powers are real and differentiable.

Exponential functions

$$\int e^x dx = e^x + C; \quad \int a^x dx = \frac{a^x}{\log a} + C,$$

$a > 0, a \neq 1.$

The special reciprocal case

$$\int \frac{dx}{x} = \log |x| + C, \quad x \neq 0.$$

Here log means the natural logarithm.

The power rule cannot be used with exponent minus one.

Quick check

$$\frac{d}{dx} (\log |x|) = \frac{1}{x}.$$

Differentiate an answer to check it.

BASIC TRIGONOMETRIC INTEGRALS

These follow directly by reversing familiar derivatives.

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \operatorname{cosec}^2 x \, dx = -\cot x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + C$$

Work on intervals where the trigonometric functions are defined. Check the minus signs in the sine and cosecant cases.

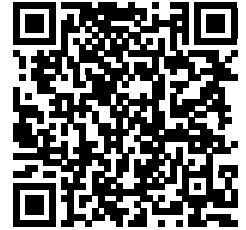


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