

MATHEMATICS • CLASS 9

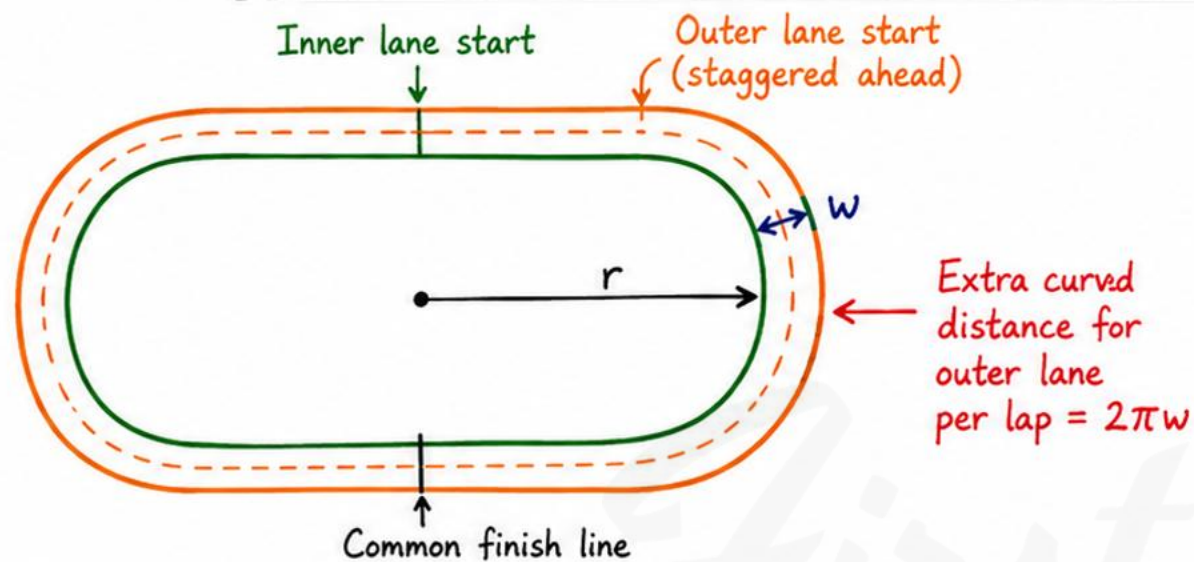
CHAPTER 6

Measuring Space Perimeter and Area

NCERT SOLUTIONS



TRACK STAGGERS — THINK AND REFLECT



Why are the starts different?

- Straight parts of all lanes have equal length.
- On bends, an outer lane has a larger radius, so its arc is longer.
- The stagger moves the outer-lane start forward to remove this extra distance.
- Therefore every runner covers the same total distance: no unfair advantage.

For one complete lap, two semicircles make one full circle:

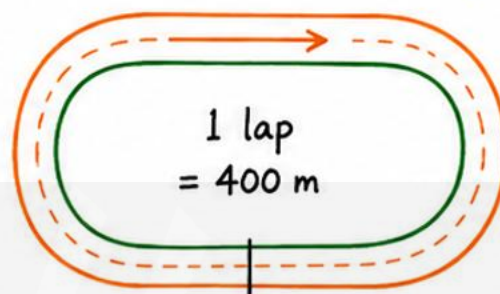
Extra distance between adjacent lanes

$$= 2\pi(r + w) - 2\pi r = 2\pi w.$$

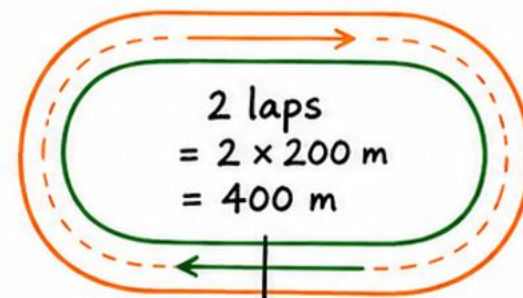
$$\text{Stagger for one lap} = 2\pi w.$$

200 m school track for the same 4 x 100 m relay

400 m track → 1 lap



200 m track → 2 laps



- The relay distance stays 400 m.
- On a 200 m track, the runners complete 2 laps.
- Extra curved distance is counted twice.

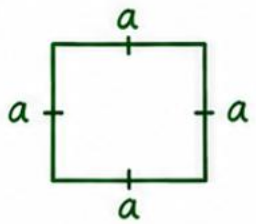
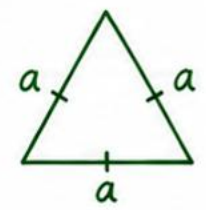
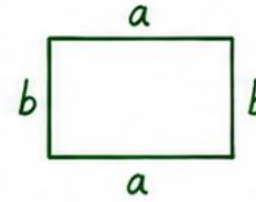
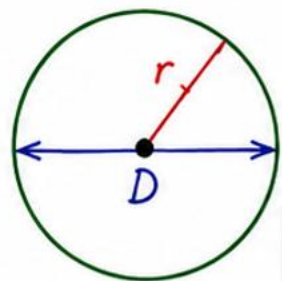
$$\text{Stagger} = 2 \times 2\pi w = 4\pi w.$$

It is not smaller. Under the same lane conditions, it is twice the one-lap stagger.

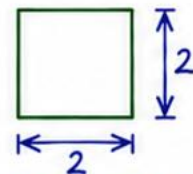
This assumes runners remain in their lanes and the finish line is common.

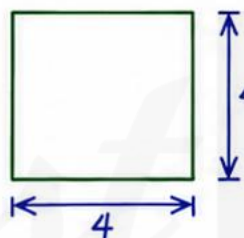
PERIMETER, SCALE AND THE C/D RATIO

① Basic perimeter

- Square side a :

 $P = 4a.$
- Equilateral triangle side a :

 $P = 3a.$
- Rectangle length a , width b :

 $P = 2(a+b).$
- Circle radius r , diameter $D = 2r$:

Its perimeter is called circumference C .
 $C = \pi D = 2\pi r.$

② Think and Reflect — scale

Square side 2
 $\Rightarrow P = 4 \times 2 = 8$

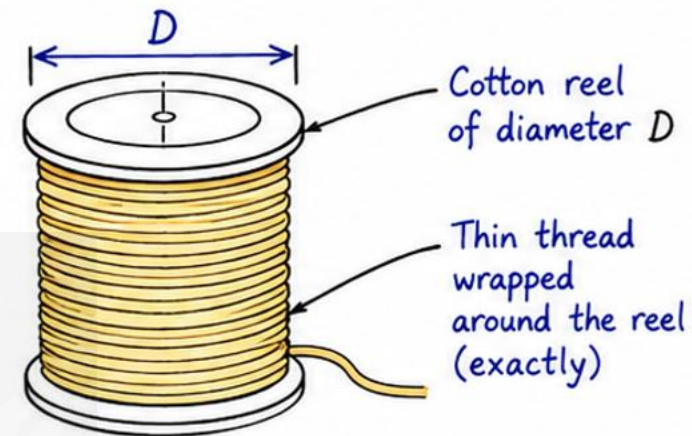
Square side 4
 $\Rightarrow P = 4 \times 4 = 16$

- Doubling every length doubles the perimeter.
- For every square, $\frac{P}{a} = 4$.
- For every equilateral triangle, $\frac{P}{a} = 3$.
- For every circle, $\frac{C}{D}$ is also constant, independent of size.

Connect to the track:

The curved parts of an athletics track are circular arcs, so their lengths come from circle circumference.

③ Home measurement of C/D



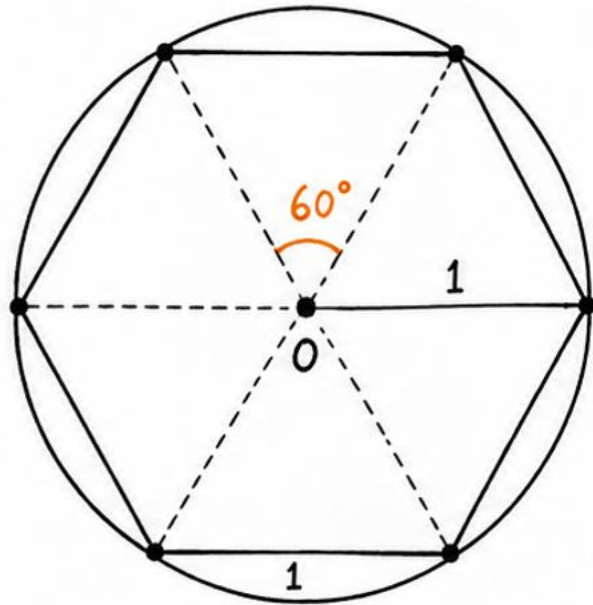
1. Measure the reel diameter D .
2. Wrap thin thread around it 20 times.
3. Unwrap and measure total thread length L .
4. One circumference $\approx \frac{L}{20}$.

Therefore $\frac{C}{D} \approx \frac{L}{20D} \approx 3.14$.

$$\frac{C}{D} = \pi, \text{ so } C = \pi D = 2\pi r.$$

WHY $3 < \pi < 2\sqrt{3}$ — HEXAGON METHOD

Inscribed regular hexagon: why $\pi > 3$

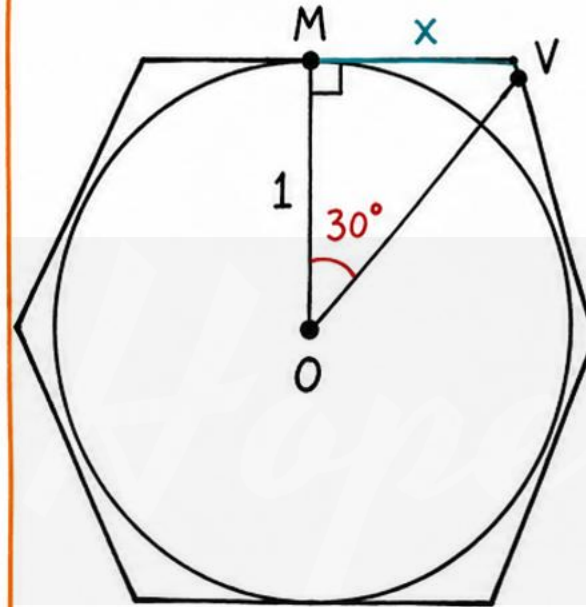


- Every central angle is 60° , so each small triangle is equilateral.
- Each hexagon side = radius = 1.
- Perimeter of hexagon = 6.
- The circle lies outside the hexagon, so circumference $C > 6$.
- Diameter $D = 2$.

$$\pi = \frac{C}{D} > \frac{6}{2} = 3.$$

$$\pi > 3.$$

Circumscribed regular hexagon: why $\pi < 2\sqrt{3}$



- In right triangle OMV, $\tan 30^\circ = \frac{x}{1} = \frac{1}{\sqrt{3}}$, so $x = \frac{1}{\sqrt{3}}$.
- Full side of hexagon = $2x = \frac{2}{\sqrt{3}}$.
- Perimeter of outer hexagon = $6 \times \frac{2}{\sqrt{3}} = 4\sqrt{3}$.
- The circle lies inside this hexagon, so $C < 4\sqrt{3}$.
- Diameter $D = 2$.

$$\pi = \frac{C}{D} < \frac{4\sqrt{3}}{2} = 2\sqrt{3} \approx 3.46.$$

$$3 < \pi < 2\sqrt{3} \approx 3.46.$$

Approximation journey of π (using only chapter facts)

Mesopotamia

3.125



Archimedes

$$3 \frac{10}{71} < \pi < 3 \frac{1}{7}$$



Zu Chongzhi

$$\frac{355}{113}$$



Āryabhaṭa

3.1416



Mādhava

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

π IS IRRATIONAL — CIRCUMFERENCE AND ARC LENGTH

π is not a fraction

- A rational number can be written as $\frac{a}{b}$, where a and b are integers and $b \neq 0$.
- Its decimal terminates or repeats.

Examples: $\frac{1}{3} = 0.333\dots$ $\frac{1}{11} = 0.090909\dots$
 $\frac{1}{7} = 0.142857142857\dots$

- $\pi = 3.1415926535\dots$ does not terminate or repeat.
- π cannot be written as a ratio of two integers, so π is irrational.

$$\pi \approx \frac{22}{7}$$

but $\pi \neq \frac{22}{7}$

$$\pi \approx \frac{355}{113}$$

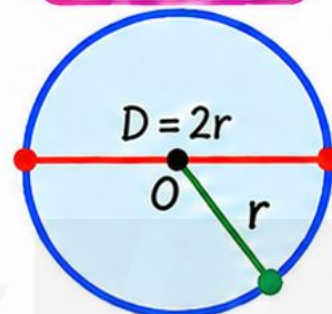
is a much closer approximation.



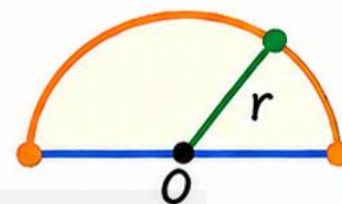
Use $\approx$ for an approximation, never $=$.

Circumference and arcs

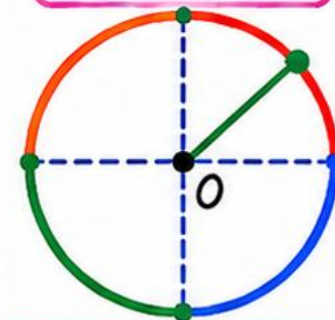
Full circle



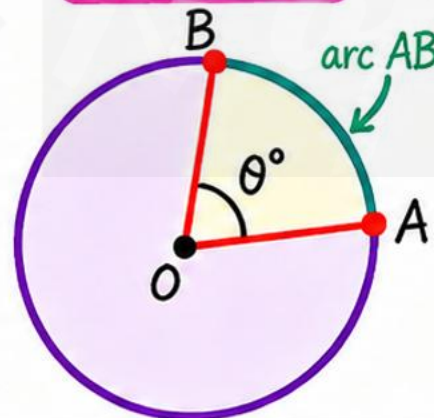
Semicircle



Quarter arcs



Sector AOB



Full circle: $C = \pi D = 2\pi r$

Semicircular arc: $\left(\frac{180^\circ}{360^\circ}\right)(2\pi r) = \pi r$

Quarter-circle arc: $\left(\frac{90^\circ}{360^\circ}\right)(2\pi r) = \frac{\pi r}{2}$

General arc AB: $l = \left(\frac{\theta^\circ}{360^\circ}\right)(2\pi r)$

Arc length = fraction of full turn $\times$ circumference.

Remember:

- Arc length does not include the two radii.
- Sector perimeter = arc length + $2r$.



FREE PREVIEW

[Click here to visit our website](#)

SCHOOL NOTES & NCERT SOLUTIONS

Complete notes and
NCERT solutions are
available on
FirstHope app only.



[Scan or click here](#)

Click here to get the app



Available on Google Play

JOIN THE FIRSTHOPE COMMUNITY

Join our Telegram channel

Click here to join Telegram



[Scan or click here](#)

@FirstHope_CBSE_Notes

Instagram

@firsthope.school

[Click here to follow >](#)

YouTube channel

@firsthope.school

[Click here to watch >](#)

FirstHope website

firsthope.co.in

[Click here to visit our website >](#)

CBSE / Class 9 / Class 10 / Class 11 / Class 12

Preview notes and NCERT solutions. Full PDFs in the FirstHope app.