

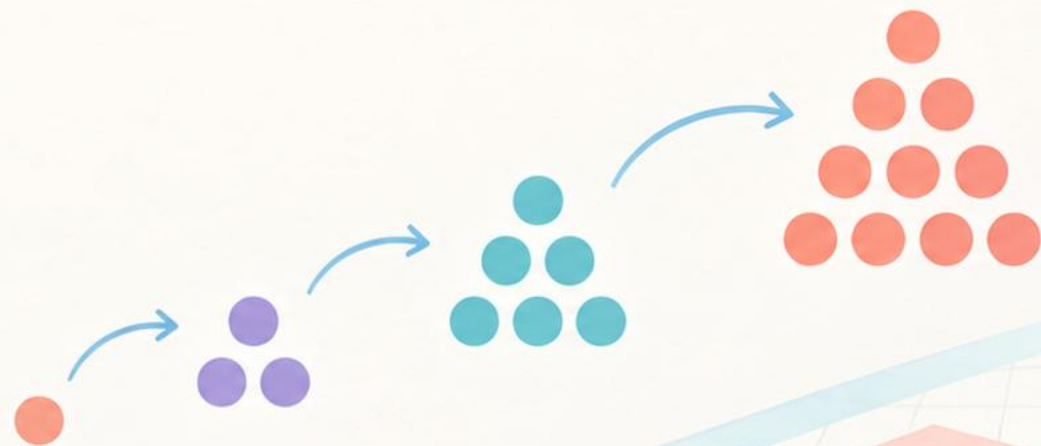
MATHEMATICS • CLASS 9

CHAPTER 8

Predicting What Comes Next

Exploring Sequences and Progressions

CHAPTER NOTES



SEQUENCES, TERMS AND NOTATION

A sequence is an ordered list of numbers. Each number is called a term.
The order matters.

Finite or infinite:

6, 12, 24, 48, 96 is a finite sequence
of five terms;

1, 2, 3, 4, 5, ... is infinite.

The three dots show that the sequence
continues indefinitely.

Position and value:

$t_1, t_2, t_3, \dots, t_n$ label successive terms.

For odd numbers, $t_1 = 1, t_2 = 3, t_3 = 5, t_4 = 7$.

In $t_4 = 7$: subscript 4 is the position;
7 is the term value.

A subscript is not an exponent. Here positions
normally start at one; some examples
explicitly start at zero.

Natural numbers	1, 2, 3, 4, 5, ...
Odd numbers	1, 3, 5, 7, 9, ...
Square numbers	1, 4, 9, 16, 25, ...
Decreasing fractions	$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$
Terms including negatives	-7, -3, 1, 5, 9, ...

Terms may be positive, negative or fractional. Not every sequence has an easy rule.

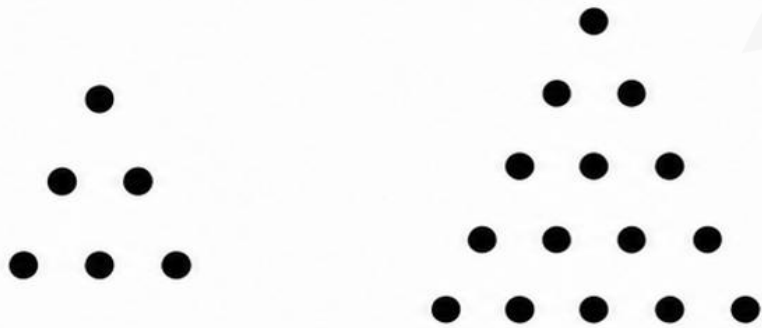
TRIANGULAR AND SQUARE-NUMBER PATTERNS

Triangular numbers:

1, 3, 6, 10, 15, ...
+2 +3 +4 +5

Add the natural numbers up to the chosen position.

$$T_n = 1 + 2 + \dots + n = \frac{n(n+1)}{2}$$



$$T_3 = 6$$

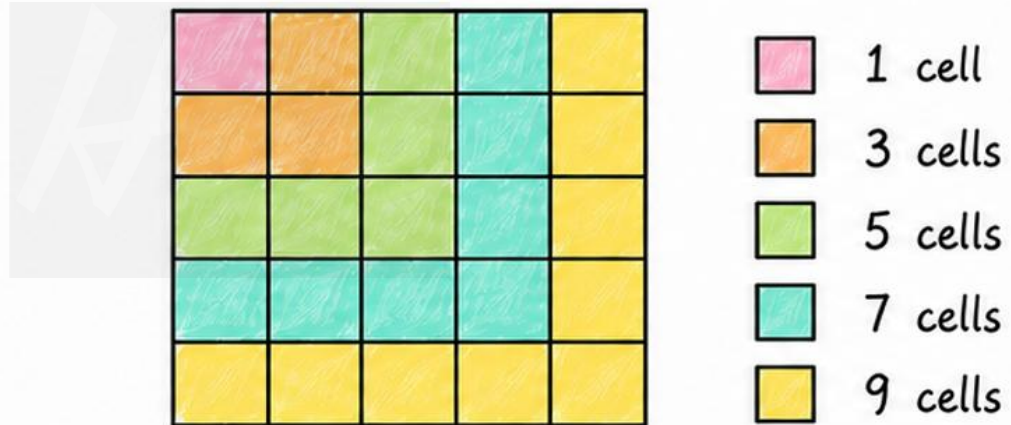
$$T_5 = 15$$

$$1 + 2 + 3 + 4 + 5 = 15$$

Square numbers:

1, 4, 9, 16, 25, ...
+3 +5 +7 +9

Consecutive odd numbers build larger squares.



$$1 + 3 + 5 + 7 + 9 = 25 = 5^2$$

$$1 + 3 + 5 + \dots + (2n - 1) = n^2$$

The n^{th} square number is the sum of the first n odd numbers.

EXPLICIT RULES AND TERM MEMBERSHIP

An explicit rule finds a term directly from its position. We do not need to calculate the earlier terms.

Odd numbers:

$$u_n = 2n - 1$$

$$u_1 = 2 \times 1 - 1 = 1$$

$$u_2 = 2 \times 2 - 1 = 3$$

$$u_3 = 2 \times 3 - 1 = 5$$

Finding a position.

$$2n - 1 = 137$$

$$\rightarrow 2n = 138$$

$$\rightarrow n = 69$$

So one hundred and thirty-seven is the sixty-ninth odd number.

Another explicit rule:

$$S_n = 5n - 2$$

first six terms

$$3, 8, 13, 18, 23, 28$$

$$S_{100} = 5 \times 100 - 2 = 498$$

$$S_{1000} = 5 \times 1000 - 2 = 4998$$

Number	Solve for n	Conclusion
308	$\frac{308 + 2}{5} = 62$	62 nd term
473	$\frac{473 + 2}{5} = 95$	95 th term
471	$\frac{471 + 2}{5} = 94.6$	not a term

A position must be a positive integer in these sequences.
A fractional position does not identify a term.

RECURSIVE RULES

A recursive rule defines a term using earlier terms. We must also know the starting term or terms.

1. Add three:

$$t_1 = 1$$

$$t_n = t_{n-1} + 3, n \geq 2$$

Sequence: 1, 4, 7, 10, 13, ...

Equivalent explicit rule:

$$t_n = 3n - 2$$

The previous term is enough.

2. Double, then add three:

$$u_1 = 1$$

$$u_n = 2u_{n-1} + 3, n \geq 2$$

$$u_2 = 2 \times 1 + 3 = 5$$

$$u_3 = 2 \times 5 + 3 = 13$$

$$u_4 = 2 \times 13 + 3 = 29$$

Next terms: 61, 125, 253.

These terms keep increasing.

$125 < 133 < 253$, so 133 is not a term.

3. Multiply by one less:

$$s_1 = 3$$

$$s_n = s_{n-1}(s_{n-1} - 1), n \geq 2$$

$$s_2 = 3(3 - 1) = 6$$

$$s_3 = 6(6 - 1) = 30$$

$$s_4 = 30(30 - 1) = 870$$

Sequence: 3, 6, 30, 870, ...

For each next term, substitute the value of the previous term into the rule.



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